# Physics and Chemistry of Sustainability II The Marvelous Side of Chemistry 

Introduction Some Historical and Basic Aspects

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75 Years $\begin{aligned} & \text { of Free Thinking. }\end{aligned}$

## What is matter? - Never mind.



Punch, or The London Charivari, London, England, 14 Jul 1855 , page 19, column 2 (filler item, anonymous) DOI: https://doi.org/10.11588/diglit.16616\#0031

## Since this lecture is part of a module in studies of science ...



M Glick (journal editor), Editorial, J Am Dent Assoc 149 (2018) 829, DOI: https://doi.org/10.1016/j.adaj.2018.08.012 see also Gordon group website / GAMESS homepage: https://www.msg.chem.iastate.edu (at the bottom)

## 1954: Advice to Students: Always think for yourself!

> Response by Limus Pauling, as spolvesann for all Mobel Laureates, to the university students of Sveden, holding a torchlight procession in the Blue Room of the City Hnll, stoekholis. 10 P.M.

Tiday, 20 Decenber 1954

When an old and distinguiahed person speoks to you, listen to him earefully and with reapect - but do not believe hing Hever put your truat in anyching but your onm intellect. Your elder, no matter whether he has oray hatr or has lost his hair, no matter whether he is a Mobel Lauronte, any be ymong. The vorld progresses, year by yenr, century by century, as the pembers of the younger generation find out what vas yrong anong the thinge that their elders sald. Fo you quat alvara be siceptical - alvnys think for yournelf.

You will have some great problems to solve - the greatest of all 1s the problem of war and peece. I belteve that this problen hes been solved, by the hydrogen bonb - that there vill never again be a world ver - the lmowledge thet a world ver vould mean vorld-wide destruetion, perhaps the end of civilisation, vill surely now lead to penament peace.

But it is your generetion that will have the job of vorking out the means of preventing disester, by inproving the techniques of international negotiations, of developing safeguanis agninst paranoic denagogues who night make nations robid; you will have this grent job to do = and I an confldent that you can do 1t.

Text: Response by Linus Pauling, as spokesman for all Nobel Laureates, to the university students of Sweden, Dec 10, 1954, Stockholm https://scarc.library.oregonstate.edu/coll/pauling/calendar/1954/12/10.html (accessed 17 Apr 2024) Photo: https://en.wikipedia.org/wiki/Linus_Pauling (CC BY-SA 2.0, accessed 17 Apr 2024)

## 1637: On Scientific Method — during Thirty Years' War (1618-1648)

DE LA METHODE
Pour bien conduirefaraifon,\&\& chercher
la verité dans lesfeiences.
PLus
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
2ui font des eflais de cete Methode.


A LExit
De limprimerie de Ian Matre.
cionocexxili Auce Priullge.

Discourse on the Method of Rightly Conducting One's Reason and of Seeking the Truth in the Sciences (anonymously published in French, "langue vulgaire", 1637)

The Method:

- Exclude all doubt (rely on evidence)
- Divide difficulties/problems into as many parts as possible
- Begin with the simplest and easiest tasks, advance to the more complex tasks
- Make enumerations as complete, reviews as general as necessary in order to be assured that nothing is omitted

Three essays included(!), one on Geometry (Cartesian axes system, foundation of Analytical Geometry)


R Descartes (R Cartesius, 1596-1650) 1619/1620 member of the armed forces of the Catholic League during the conquest of Prague
see also: Bibliothèque Nationale de France (1st edition $>$ )
"Common sense is the best distributed thing in the world, for everyone thinks being so well endowed with it that even those who are the most difficult to satisfy in every other way are not in the habit of desiring more of it than what they have." (English translation of 1st sentence, 1st paragraph of Part I)

Title page: https://en.wikipedia.org/wiki/Discourse_on_the_Method (public domain, accessed 17 Apr 2024) Portrait: https://en.wikipedia.org/wiki/René_Descartes (painting by F Hals, public domain, accessed 17 Apr 2024)

## 1631: Magdeburg's Sacrifice



$\Delta$ Number of Inhabitants of Magdeburg vs. Time (1400-1870)

## Magdeburg

One of the biggest German cities at the end of the Middle Ages 1597 the plague took 10.000 lifes
¢ 1631 Magdeburg's Sacrifice
1639 only 450(!!) inhabitants 1683 the plague took 2.650 lifes

May 20-24 ${ }^{\text {greg, }}$ 1631: Magdeburg's Sacrifice (also called "Magdeburg's Wedding")
Destruction of the Protestant city by the Imperial Army and its allied forces (Catholic League); less than 4.000 of the $\sim 25.000$ inhabitands survived (worst massacre of Thirty Years' War).

Picture: Engraving by D Manasser, 1632, https://en.wikipedia.org/wiki/Sack_of_Magdeburg (public domain, accessed 17 Apr 2024) Diagram: https://de.wikipedia.org/wiki/Einwohnerentwicklung_von_Magdeburg (CC0, accessed 17 Apr 2024)

## 1650-1680: The Magdeburg hemispheres and the Boyle-Mariotte law



O von Guericke (1602-1686, until 1666 O Gericke) - Fled from his birth town before 1631, lost all his personal property when the city was incinerated during Magdeburg's

## Sacrifice




R Boyle (1627-1692) — Learned about von Guericke's work from the book by Schott; experimental study of air together with R Hooke (Hooke's law)

The Magdeburg hemispheres (1654) - a tool to demonstrate the force of atmospheric pressure - empirical proof of the existence of vacuum

Boyle's law or Boyle-Mariotte law (1662/1676, named after R Boyle and E Mariotte):

$$
p V=\text { const } \quad(\text { pressure } p, \text { volume } V, m \text { const }, T \text { const })
$$

Hyperbolas in a $p$ - $V$-diagram ( $\downarrow$, note the use of Cartesian axes)
Portrait of von Guericke: https://en.wikipedia.org/wiki/Otto_von_Guericke (picture by A van Hulle, public domain, accessed 17 Apr 2024)
Picture: From a book by G Schott, 1657, https://en.wikipedia.org/wiki/Magdeburg_hemispheres (public domain, accessed 17 Apr 2024) Portrait of Boyle: Oil painting by J Kerseboom, 1689, https://de.wikipedia.org/wiki/Robert_Boyle (public domain, accessed 17 Apr 2024)

- Science develops further during times of crisis (war times, times with difficulties in local/regional/national economy)
- But are crises really always necessary for (accelerating) the advancement of science?
- Can a better/good/excellent knowledge of science (maths/physics/chemistry/...) for everyone help?


Supertramp - Crisis? What Crisis? (1975) (album front cover, (c)A \& M Records)

## An almost perfectly sustainable application of an inert gas

Boyle-Mariotte law:

$$
p V=\text { const }
$$

( $T$ constant, $m$ constant)

> Gas spring
(typical filling gas: $\mathrm{N}_{2}$, typical pressure range:
10... 200 bar)

Other applications: shock absorbers, air-filled rubber tyres

$$
\begin{aligned}
& 1 \text { - Piston rod } \\
& 2 \text { - Head cap } \\
& 3 \text { - Piston rod wiper } \\
& 4 \text { - Piston rod guide bushing } \\
& 5 \text { - Retaining ring } \\
& 6 \text { - O-ring } \\
& 7 \text { - Piston rod seal } \\
& 8 \text { - Cylinder } \\
& 9 \text { - Piston } \\
& 10 \text { - Flow-restriction orifice } \\
& 11 \text { - Piston guide bushing } \\
& 12 \text { - Valve } \\
& 13 \text { - Valve-sealing screw }
\end{aligned}
$$

## 1713: Sustainability and Forestry

## First-time appearance of "sustain. .." in written form in Europe

Around 1700, the mining industry in the Ore Mountain region (Saxony \& North Bohemia), at that time one of the largest mining regions in Europe, was threatened. It was not that the mines had been exhausted of their ores, the problem was an acute scarcity of timber. (Wikipedia)

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 Dartinenberuben/ wie eine fothane Confervation unt 2 2 nbau des Soltes ansufelleth dases cite continuirlide ebeftandige unt nady baltende sukutg gebe/ weiln es cine unentberlid)e Sade ift/obne D welde

HC von Carlowitz (1645-1714): Sylvicultura oeconomica (1713, cover page and part of p 105)
Commemorative plaque, created by B Freiesleben, at a house in Freiberg (Saxony, Germany)
Picture of plaque: https://de.wikipedia.org/wiki/Hans_Carl_von_Carlowitz (CC BY 3.0, accessed 17 Apr 2024)
Title page and text passage: de.wikipedia.org/wiki/Nachhaltigkeit_(Forstwirtschaft) (public domain, accessed 17 Apr 2024)

## What is the meaning of words?

nachhalten (verb) - to last (to last for a longer time)
nachhaltig (adj.) - effective, enduring, (long) lasting, sustained, sustainable
nachhaltige Wirkung - (long) lasting effect
nachhaltig wirken - to have a (long) lasting effect
nachhaltige Bemühungen - sustained efforts
nachhaltige Entwicklung - sustainable development
Nachhaltigkeit - sustainability(?), longlastingness(?), effectiveness, efficiency

- "Sustainability" did link - from its beginning - economic and ecological aspects (and still does)!
- Remember: Deforestation occurred due to mining and smeltering activities in the Ore Mountain region
- But: Trees don't grow on/from wood chips or on/from recovered paper!
- Today we now that carbon dioxide, $\mathrm{CO}_{2}$, taken from the air, is the main source for carbon for all plants (all forms of life based on or using photosynthesis)
- Even today, deforestation is still a highly important issue on the global scale


## The Keeling Curve



FIGURE 0-4 Monthly average atmospheric $\mathrm{CO}_{2}$ measured on Mauna Loa. This graph, known as the Keeling curve, shows seasonal oscillations superimposed on rising $\mathrm{CO}_{2}$ Data from hitp://scrippscoz.ucsd.edu/date) in_stu_co2/monthly_mlo.csv.


D Harris, Quantitative Chemical Analysis, 8th ed, Freeman, New York, 2010, pp 1 \& 4

Q: How much is 400 ppm of $\mathrm{CO}_{2}$ ?
Reminder: Assuming volume ratio, $V / V, 1 \mathrm{ppm}$ corresponds to 1 mL in $1 \mathrm{~m}^{3}$ $\left(1 \mathrm{~m}^{3}=1000 \mathrm{~L}=10^{6} \mathrm{~mL}\right)$

## SUSTAINABLE DEVELOPMENT <br> G*ALS



United Nations General Assembly: Transforming our World: the 2030 Agenda for Sustainable Development, 25 Sep 2015 Source: https://www.un.org/sustainabledevelopment/news/communications-material/ (accessed 17 Apr 2024)

- EN: Sustainable Development Goals (SDGs) DE: Ziele für nachhaltige Entwicklung FR: Objectifs de développement durable
- Most of these 17 SDGs (perhaps all of them) are very intimately linked to, or connected with chemistry and physics
- Public discussions / the daily/weekly news, if associated with these 17 SDGs, seem to largely ignore physics- and chemistry-related aspects
- Can we safely assume being well on the right track if basic knowledge in science gets/is/remains out of scope?
- Chemistry and physics provide important information for what one may like to call "the frame in which our life takes place", or "the stage on which our life evolves"


## 1 - No Poverty

- Matter-related and energy-related aspects of poverty
- Enough money to pay for coverage of the most elementary needs related to food,
clothing, lodging,
heating and/or cooling,
media supply (e.g. water, electricity, natural gas)


## 2 - Zero Hunger

- Sufficient supply of proteins, carbohydrates, lipids, micro nutrients (vitamins, trace elements),
- Food quality guaranteed?
- Food source?
- Agricultural production guaranteed?
- Transportation of food guaranteed?
- Distribution/delivery of food guaranteed?


## 3 - Good Health and Well-Being

- Matter- and energy-related conditions of good health and well-being
- Molecular Medicine
- Molecular Biology / Biochemistry
- ... but not only "bio":
- 1910 salvarsan (cyclo-(RAs) $n, \mathrm{R}=3$-amino-4-hydroxyphenyl, $n=3$ or 5 , antimicrobial drug, effective to treat e.g. syphilis)

- 1978 cisplatin ([Pt( $\left.\left.\mathrm{NH}_{3}\right)_{2} \mathrm{Cl}_{2}\right]$, anticancer drug)
- Access to good-quality medicine / dentistry


## 4 - Quality Education



- Quality of education in STEM (science, technology, engineering, mathematics), emphasizing the great importance of these subjects for the well-being of us humans as well as of other forms of life on this planet


## 5 - Gender Equality

- Assuming some sort of connection between biological sex and gender
- Estradiol - a steroid hormone, likely the most important sex hormone in mammals (both female and male individuals), but also found in other animal species, e. g. fish, crustaceans, insects


estradiol, $\mathrm{C}_{18} \mathrm{H}_{24} \mathrm{O}_{2}(M=272.38 \mathrm{~g} / \mathrm{mol})$
structural formula with stereochemical information (left) and ball-and-stick representation (right)


## 6 - Clean Water and Sanitation

- Water

- Water quality:
- drinking water
- raw water
- fresh water / brackish water / saline water (oceans) / brine water (Dead Sea)
- Chemical (physical / biological) water treatment
- Water supply network / Sewage water network (material of pipes, canals, etc.)
- Cleaning agents (detergents, tensides)
- Disinfection agents (silver salts / chlorine-based / alcohol-based / ...)


## 7 - Affordable and Clean Energy

$\frac{11}{(0)}$

- Which source of energy for which purpose (household / transportation on the road, by railway, by air freight, by ship / ...)
- Solar power? Fuel cell? Electric battery? Combustion engine (hydrogen/CNG/LPG/gazoline/oil/coal)?
- Side products and waste during production process and under operating conditions?


## 8 - Decent Work and Economic Growth

- No material system can grow indefinitely $(t \rightarrow \infty)$ and unboundedly $(R \rightarrow \infty)$ in a finite world (planet Earth / solar system)


## 9 - Industry, Innovation and Infrastructure

- Any sort of material (matter) processed by the producing industry has some chemical composition


## 10 - Reduced Inequalities

- No (direct) link to Chemistry - Really?


## 11 - Sustainable Cities and Communities

- How to construct buildings? Concrete, bricks, steel, glass, wood, . . .?
- Urban Mining (old buiuldings, which are considered no longer useful, serve as sources of material to construct new ones)


## 12 - Responsible Consumption and Production

- Responsibility with respect to whom?
- Beware of entropy!


## 13 - Climate Action

- Keeling curve (see above)
- Other greenhouse gases $\left(\mathrm{N}_{2} \mathrm{O}, \mathrm{CH}_{4}, \ldots\right)$


## 14 - Life below Water

- Does this really need a hint to recognize its connection to (Bio-)Chemistry?


## 15 - Life on Land (and in the Sky)



- Does this really need a hint to recognize its connection to (Bio-)Chemistry?


## 16 - Peace, Justice and Strong Institutions

- No (direct) link to Chemistry - Really?
- Chemical Weapons?
- Organization for the Prohibition of Chemical Weapons (OPCW)


## 17 - Partnerships for the Goals

- No (direct) link to Chemistry - Really?

Our "Construction Kit": The Periodic Table of the Chemical Elements ...

$\ldots$ is built up from "blocks" of width $4 \ell+2=2(2 \ell+1)$ :
s block $(\ell=0)$, p block $(\ell=1)$, d block $(\ell=2)$, f block $(\ell=3)$

## Abundance of the chemical elements in the Earth's crust



$$
\operatorname{Only}(\mathrm{O})>\operatorname{Strong}(\mathrm{Si})>\operatorname{Athletes}(\mathrm{Al})>\ln (\mathrm{Fe})>\operatorname{College}(\mathrm{Ca})>\operatorname{Study}(\mathrm{Na})>\operatorname{Past}(\mathrm{K}) \approx(\text { ? }) \text { Midnight }(\mathrm{Mg})
$$

Source: https://en.wikipedia.org/wiki/Abundance_of_the_chemical_elements (public domain, accessed 17 Apr 2024)


European Chemical Society, 2018, https://www.euchems.eu/wp-content/uploads/2018/10/Periodic-Table-ultimate-PDF.pdf (accessed 17 Apr 2024)


European Chemical Society, 2021, https://www.euchems.eu/wp-content/uploads/2021/11/Endangered-ElementsCarbon-Updated.pdf
(accessed 17 Apr 2024)


European Chemical Society, 2023, https://www.euchems.eu/wp-content/uploads/2023/06/PT2023_protected.pdf (accessed 17 Apr 2024)

List of countries and territoires by area / by population (2022)

| Country/Territory | Total area $/ \mathrm{km}^{2}$ |
| :--- | ---: |
| Russia | $17,098,246$ |
| Antarctica | $14,200,000$ |
| Canada | $9,984,670$ |
| China | $9,596,960$ |
| United States | $9,525,067$ |
| Brazil | $8,510,346$ |
| Australia | $7,741,220$ |
| European Union | $4,242,351$ |
| India | $3,287,263$ |
| Total surface area | $510,072,000$ |


| Country/Territory | Human population |
| :--- | ---: |
| China | $1,409,670,000$ |
| India | $1,392,329,000$ |
| European Union | $450,359,450$ |
| United States | $335,893,238$ |
| Indonesia | $279,118,866$ |
| Pakistan | $241,499,431$ |
| Nigeria | $223,800,000$ |
| Brazil | $203,080,756$ |
| Bangladesh | $169,828,911$ |
| Russia | $146,203,613$ |
| Sum | $4,851,783,265$ |
| World Population (2022) | $7,975,105,156$ |

To be kept in mind:

- Surface area provides space e.g. for cultivation of food crops, for operation of power plants (solar, wind, hydrothermal, fossil, nuclear, ...). Surface area has resources (composed of around 80 different chemical elements) on surface or covers them underground.
- Every single human being wants her/his most basic needs (e.g. water, food, warmth, light) be met and satisfied, day by day.


## History and Basics

- 1713 - HC von Carlowitz (Saxony): Sustainability and Forestry
- 1789 - Lavoisier (France, with support by his wife): Elements; Law of conservation of mass $(\Delta m=0)$ for all chemical reactions
- 1793 - J Beckmann (professor at Göttingen): "Warenkunde" (to know where to find and how to handle commercial goods, how to probe their quality); coined the term "technology"


1784/1795 - I Kant (22.4.1724-1804)

1784: Essay Answer to the Question "What is Enlightenment?" (German: Was ist Aufklärung?)
Answer: "Enlightenment is man's emergence from his self-incurred immaturity (Unmündigkeit)."
Sapere aude! (actually a motto from Horace) - $\mathfrak{F a b e} \mathfrak{P u t h}$, did deinez eigenen $\mathfrak{Z e r i t a n d e z}$ zu bedienen! - Dare to be wise! - Ose penser par toi-même!

1795: Essay Perpetual Peace (German: Zum ewigen Frieden)

## April 22: Earth Day

Annual event (since 1970) to demonstrate support for environmental protection


Unofficial Earth Day Flag, created by John McConnell (1915-2012), includes The Blue Marble photograph taken by the crew of Apollo 17 on Dec 7 , 1972

This year's Earth Day official theme is "Planet vs. Plastics"

- 1810 - J Dalton (England): Modern idea of the atom; Stoichiometry ("measuring the elements")
- 1811 - A Avogadro (Italy): Same volumes of different gases contain same number of gas particles ( $p, T$ both constant)
- 1834 - E Clapeyron (France): first formulation of the ideal gas law $p V=n R T$
- 1860 - Karlsruhe Congress (Germany): First international meeting of chemists; Avogadro's hypothesis accepted as correct; adoption of a common set of atomic weights
- 1869 - D Mendeleev (Russia): First version of the Periodic Table of the Elements
- 1960-1 mol of $X$ has as many particles of type $X$ as there are atoms in 12 g of carbon isotope ${ }^{12} \mathrm{C}$
- 2019-1 mol of $X$ contains exactly(!) $6.02214076 \cdot 10^{23}$ particles of type $X$

Avogadro's constant (now exactly fixed):

$$
N_{A}=6.02214076 \cdot 10^{23} / \mathrm{mol}
$$

- 1835 - WR Hamilton (Ireland): New formulation of Classical Mechanics; Hamilton's function $H$ gives total energy as sum of kinetic and potential energies, $T$ and $V$ :

$$
H=T+V
$$

Freely moving particle (mass $m$, speed $v$ ): $T=(1 / 2) m v^{2}$
Scaling laws:
$m \rightarrow 2 m$ (car, $1000 \mathrm{~kg} \rightarrow 2000 \mathrm{~kg}$ ) gives $T \rightarrow 2 T$, but $v \rightarrow 2 v($ car, $100 \mathrm{~km} / \mathrm{h} \rightarrow 200 \mathrm{~km} / \mathrm{h})$ gives $T \rightarrow 4 T$ !

- 1842/1847 - R Mayer \& H von Helmholtz (Germany): Law of conservation of energy ( $\Delta E=0$ ) in all (bio-)physical processes
- 1865 - R Clausius (Germany): Introduced entropy (S); in all observable processes, the system's entropy never decreases ( $\mathrm{d} S \geq 0$ )
- 1864 - CM Guldberg \& P Waage (Norway): Law of mass action, e.g. for a chemical reaction

$$
a \mathrm{~A}+b \mathrm{~B} \rightleftharpoons x \mathrm{X}+y \mathrm{Y}
$$

at equilibrium (temperature $T$, molar concentration of $\mathrm{A}[A]$ etc.)

$$
K=\frac{[A]^{a}[B]^{b}}{[X]^{x}[Y]^{y}}=\text { const }
$$

- 1884/1888 - H Le Chatelier (France): A system in equlibrium, which is put under stress in some way, tries to evade that stress as much as possible when it adopts a new equilibrium (Le Chatelier's principle) 2 examples (Haber-Bosch ammonia synthesis, $\mathrm{NO}_{2}$ equilibrium):

$$
\mathrm{N}_{2}+3 \mathrm{H}_{2} \rightleftharpoons 2 \mathrm{NH}_{3} \quad, \quad 2 \mathrm{NO}_{2 \text { red-brown }} \rightleftharpoons \mathrm{N}_{2} \mathrm{O}_{4 \text { colorless }}
$$

increase of pressure increases the amount of reaction product (requiring less volume)

- 1943 - E Schrödinger (Dublin): What is Life?

Q1: What is the physico-chemical basis behind genetic inheritance?
A1: Genetic information is contained in an "aperiodic crystal"
Q2: What is going on during metabolism?
A2: A living organism succeeds, during all its life, from freeing itself from the entropy that it cannot avoid to produce while it is alive.

What is Life? - For an animal, like us humans, living under aerobic conditions ...

- breathing $\left(\mathrm{O}_{2} \downarrow, \mathrm{CO}_{2} \uparrow\right)$
- drinking $\left(\mathrm{H}_{2} \mathrm{O}\right) \&$ eating (proteins, carbohydrates, lipids, fibres, ...)
- clothing (wool, cotton, nylon, ...)
- housing (wood, brick, ...)
- moving (by yourself?, by vehicle?, energy source?)
- reproducing (genetic code)


[^0]- 1943 - A Maslow (USA): hierarchy of needs

- 1945 - K Popper (New Zealand): The Open Society and Its Enemies A democratic system is a system where the change of government happens without spilling of blood
- 1980 - The Global 2000 Report to the President (commissioned by J Carter, large ignored by his successor R Reagan)
- 1983/1987 - Brundtland Commission (UN) \& Brundtland Report ("Our Common Future") contained a definition of the term "sustainable development" "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs."
- 2011 - R Kümmel (professor Theoretical Physics, Würzburg): The Second Law of Economics on the role / importance of entropy for economy


## 1998: Green Chemistry

- Prevention
- Atom Economy
- Less Hazardous Chemical Syntheses
- Designing Safer Chemicals
- Safer Solvents and Auxiliaries
- Design for Energy Efficiency
- Use of Renewable Feedstocks
- Reduce Derivatives
- Catalysis
- Design for Degradation
- Real-time Analysis for Pollution Prevention
- Inherently Safer Chemistry for Accident Prevention

Need to always think Chemistry together with Engineering $\rightarrow$ Green Chemistry \& Green Engineering

P Anastas, J Warner: Green Chemistry - Theory and Practice, Oxford University Press, 1998, p 30 https://www.acs.org/greenchemistry/ (accessed 17 Apr 2024)

1998/2002: Cradle to cradle (C2C) — instead of "cradle to grave" CRADLE TO CRADLE
A concept by Michael Braungart and William McDonough


Why so late? (after more than 250 years of development in science)
This cannot work literally $100 \%$, there is always some loss

www.amazon.com, www.amazon.de

1920 - Fritz Haber - The Recycling Principle
Was an einer Stelle der natürlichen Roh- stoffbasis durch die Technik entnommen würde, müsse nach seiner wirtschaftlichen Nutzung wieder zur Quelle zurückkehren, damit das ursprüngliche Material ?in neue Formen gebracht aufs Neue nutzbar wird?, wie er damals schrieb und wie sich als immer wichtiger herausstellt, seit- dem Menschen sich um Nachhaltigkeit bemühen.
quoted from EP Fischer: Ein Scheiterhaufen der Wissenschaft, Springer, Berlin, 2023, p. 55

Life Cycle analysis/assessment (LCA) - actually only Life Span Analysis, at best see lecture by Christoph Tzschucke

## The Language of Change

For countable/measurable stuff:

Q: How to describe changes of amount of that stuff over time? How to describe changes/differences in space (in position)?

A: Set up and (try to) solve differential equations!

Ordinary Differential Equations (ODEs)
for $f(x)$ or $f(t)$
Partial Differential Equations (PDEs)
for $f(\underline{x})$ or $f(\underline{x}, t)$


Astérix: Le Domaine des Dieux (English: The Mansions of the Gods, 1973; German: Die Trabantenstadt, 1974), comic book series, vol. 17, p. 15

> People think that mathematics is complicated. Mathematics is the simple bit, it's the stuff we CAN understand. It's cats that are complicated.
> John Horton Conway


John H. Conway (1937-2020), known for "Conway's Game of Life" (1968)

Picture: R. T. Curtis, Biographical Memoirs of Fellows of the Royal Society 72 (2022) 117, DOI:10.1098/rsbm.2021.0034 https://en.wikipedia.org/wiki/Cellular_automaton

Hyperbolic growth
The logistic curve
Lotka-Volterra model
Daisyworld (Lovelock \& NN)

## Some simple (and naïve) growth laws

- Exponential growth/decay ( $k$ growth/decay constant, SI unit $[k]=s^{-1}$ ):

Differential equation:

$$
\begin{gathered}
\dot{Q}=\frac{\mathrm{d} Q}{\mathrm{~d} t}=k Q \\
\dot{Q}-k Q=0
\end{gathered}
$$

Integrated forms:

$$
Q(t)=Q_{0} \exp \left[k\left(t-t_{0}\right)\right] \quad\left(Q_{0}=Q\left(t_{0}\right)\right)
$$

- linearized:

$$
\ln \left(\frac{Q(t)}{Q_{0}}\right)=k\left(t-t_{0}\right)
$$

$k>0$ : exponential growth, doubling time $T_{2}=\ln (2) / k$ (time interval for increase from $Q_{0}$ to $2 Q_{0}$; it took infinitely long to reach $Q_{0}$ at time $t_{0}$; "shortening of the future")
$k<0$ : exponential decay, "half life" $T_{1 / 2}=\ln (1 / 2) / k=-\ln (2) / k$ (time interval for decrease from from $Q_{0}$ to $Q_{0} / 2$; it takes infinitely long for $Q_{0}$ to vanish)

## Some simple (and naïve) growth laws (contd)

Exponential growth $(k>0)$

doubling time $T_{2}=\ln (2) / k$
(took infinitely long to reach $Q_{0}$ at time $t_{0}$ )

Exponential decay $(k<0)$

"half life" $T_{1 / 2}=\ln (1 / 2) / k=-\ln (2) / k$
(takes infinitely long for $Q_{0}$ to vanish completely)

No finite real system can actually follow such behaviour over an arbitrarily long period of time!

## Some simple (and naïve) growth laws (contd)

- Limited growth/decay ( $k>0$ rate constant, SI unit $[k]=s^{-1}$, constant limiting size $S>0$ ): Differential equation:

$$
\begin{aligned}
& \dot{Q}=k(S-Q) \\
& \dot{Q}+k Q=k S
\end{aligned}
$$

Integrated forms:

$$
Q(t)=S-\left(S-Q_{0}\right) \exp \left[-k\left(t-t_{0}\right)\right] \quad\left(Q_{0}=Q\left(t_{0}\right), S=\lim _{t \rightarrow \infty} Q(t)\right)
$$

- linearized:

$$
\ln \left(\frac{Q(t)-S}{Q_{0}-S}\right)=-k\left(t-t_{0}\right)
$$

The deviation from $S=\lim _{t \rightarrow \infty} Q(t)$ vanishes exponentially
But $Q(t)$ diverges as we look into the past, i. e. when $t \rightarrow-\infty$ (diverges either to $+\infty$ or to $-\infty$ )
$Q<S \Leftrightarrow \dot{Q}>0$ : growth towards limiting size $S$ ("saturation limit", "system capacity")
$Q>S \Leftrightarrow \dot{Q}<0$ : decay towards limiting size $S$ ("saturation limit", "system capacity")
Important question: What is (the best/right/correct value for) $S$ ?
hyperbolic growth (explosion in finite time)

Heinz von Foerster (1911-2002), Biological Computer Lab, University of Illinois, 1963 https://en.wikipedia.org/wiki/Heinz_von_Foerster CC BY-SA 4.0
Sebastian von Hoerner (1919-2003)
https://www.nrao.edu/archives/items/show/29914
L. Roberts, 9 Billion?, Science 333 (2011) 540 (Link)

## Chemical kinetics (some instructive examples)

$A, B, \ldots$
$n$ different types of stuff . . . countable (e.g. individuals) or measurable (mass, amount of substance), alternatively: densities of that stuff (number/amount per area/volume, assuming area/volume to be constant, i.e. time-independent) - assuming differentiability (even in case of countable stuff)
$k>0, k_{i}>0$
rate constants, elements of a singular non-symmetric $n \times n$ rate constant matrix $\underline{\underline{K}}$ (has no inverse $\underline{\underline{K}}^{-1}$ )
Coupled set of first-order ODEs with initial conditions (conditions at time $t=t_{0}$ ):

$$
\frac{\mathrm{d}}{\mathrm{~d} t}\left(\begin{array}{c}
A  \tag{1}\\
B \\
\vdots
\end{array}\right)=\underline{=}\left(\begin{array}{c}
A \\
B \\
\vdots
\end{array}\right) \quad \text { with }\left.\quad\left(\begin{array}{c}
A \\
B \\
\vdots
\end{array}\right)\right|_{t_{0}=0}=\left(\begin{array}{c}
A_{0} \\
B_{0} \\
\vdots
\end{array}\right)
$$

This has solutions of form $\underline{x} \mathrm{e}^{\lambda t}$ with $(\lambda, \underline{x})$ obtained from the - possibly generalized ( $p \geq 2$, see below) eigenvalue problem of $\underline{\underline{K}}$ : Denote $\underline{\underline{X}}=\left(\underline{x}_{1}, \ldots, \underline{x}_{n}\right)$ and $\underline{\underline{\Lambda}}=\operatorname{diag}\left(\lambda_{i}, \ldots, \lambda_{n}\right)$, then

$$
\left(\underline{\underline{K}}-\lambda_{i} \underline{\underline{\mathrm{E}}}\right)^{p} \underline{x}_{i}=\underline{o} \quad \Rightarrow \quad \text { if } p=1: \quad \underline{\underline{K}} \underline{\underline{X}}=\underline{\underline{X}} \underline{\underline{\Lambda}} \Leftrightarrow \underline{\underline{X}}^{-1} \underline{\underline{K}} \underline{\underline{X}}=\underline{\underline{\Lambda}}
$$

If $\lambda_{i} \neq \lambda_{j}$ for all $i \neq j$, the complete solution of eq. (1) is:

$$
\left(\begin{array}{c}
A \\
B \\
\vdots
\end{array}\right)=\sum_{i} c_{i} \underline{x}_{i} \mathrm{e}^{\lambda_{i} t} ; \quad \text { initial condition } \quad\left(\begin{array}{c}
A_{0} \\
B_{0} \\
\vdots
\end{array}\right)=\sum_{i} c_{i} \underline{x}_{i}=\underline{\underline{X}}\left(\begin{array}{c}
c_{1} \\
\vdots \\
c_{n}
\end{array}\right) \quad \text { gives }\left(\begin{array}{c}
c_{1} \\
\vdots \\
c_{n}
\end{array}\right)=\underline{X}^{-1}\left(\begin{array}{c}
A_{0} \\
B_{0} \\
\vdots
\end{array}\right)
$$

## Chemical kinetics (some instructive examples, contd.)

I. $A \xrightarrow{\mathrm{k}} \mathrm{B}$
two compounds, a single reaction (only forward)
II. $B \stackrel{k_{1}}{\longleftrightarrow} A \xrightarrow{k_{2}} C$
three compounds, two parallel reactions (both only forward)
III. $A \xrightarrow{k_{1}} B \xrightarrow{k_{2}} C$
three compounds (or resource/product/waste), two consecutive reactions (both only forward)
IV. $A \underset{k_{2}}{\stackrel{k_{1}}{\rightleftharpoons}} B$
two compounds, two reactions (forward and reverse) - or a 2-loop
$\mathbf{V} \ldots \xrightarrow{\mathrm{k}_{3}} A \xrightarrow{\mathrm{k}_{1}} B \xrightarrow{\mathrm{k}_{2}} C \xrightarrow{\mathrm{k}_{3}} A \xrightarrow{\mathrm{k}_{1}} \ldots$
three compounds (or resource/product/waste), three consecutive reactions (all only forward), cyclic - or a 3-loop

All good things come in threes. ( $\mathrm{A}, \mathrm{B}, \mathrm{C}$ )

## Chemical kinetics (some instructive examples, contd.)

I. $A \xrightarrow{k} B$

$$
\begin{aligned}
& \dot{A}= \\
& \dot{B}=
\end{aligned} \quad-k A \quad k A \quad \equiv\left\{\frac{\mathrm{~d}}{\mathrm{~d} t}\binom{A}{B}=\underline{\underline{K}}\binom{A}{B}, \quad \underline{\underline{K}}=\left(\begin{array}{cc}
-k & 0 \\
k & 0
\end{array}\right)\right.
$$

Solving the eigenvalue problem of $\underline{\underline{K}}$ :

$$
\underline{\underline{X}}^{-1} \underline{\underline{K}} \underline{\underline{X}}=\underline{\underline{X}}^{-1} \underline{\underline{K}}\left(\underline{x}_{1}, \underline{x}_{2}\right)=\left(\begin{array}{ll}
1 & 0 \\
1 & 1
\end{array}\right)\left(\begin{array}{cc}
-k & 0 \\
k & 0
\end{array}\right)\left(\begin{array}{cc}
1 & 0 \\
-1 & 1
\end{array}\right)=\left(\begin{array}{cc}
-k & 0 \\
0 & 0
\end{array}\right)=\operatorname{diag}(-k, 0)=\operatorname{diag}\left(\lambda_{1}, \lambda_{2}\right)=\underline{\underline{\Lambda}}
$$

Solving the chemical kinetics problem:

$$
\binom{A}{B}=A_{0}\binom{1}{-1} \mathrm{e}^{-k t}+\left(A_{0}+B_{0}\right)\binom{0}{1} \quad \xrightarrow{t \rightarrow \infty} \quad \lim _{t \rightarrow \infty}\binom{A}{B}=\binom{0}{A_{0}+B_{0}}
$$

or component-wise:

$$
A(t)=A_{0} \mathrm{e}^{-k t} ; \quad B(t)=A_{0}\left(1-\mathrm{e}^{-k t}\right)+B_{0}
$$

At the end (of time), compound $A$ has completely vanished and only compound $B$ is present, in total amount $\mathrm{A}_{0}+\mathrm{B}_{0}$.

## Chemical kinetics (some instructive examples, contd.)

II. $B \stackrel{k_{1}}{\longleftrightarrow} A \xrightarrow{k_{2}} C$

$$
\left.\begin{array}{rrrr}
\dot{A}= & -k_{1} A-k_{2} A & = & -\left(k_{1}+k_{2}\right) A \\
\dot{B}= & k_{1} A & = & k_{1} A \\
\dot{C}= & k_{2} A & = & k_{2} A
\end{array}\right\} \equiv\left\{\frac{\mathrm{d}}{\mathrm{~d} t}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=\underline{=}\left(\begin{array}{c}
A \\
B \\
C
\end{array}\right), \quad \underline{=}=\left(\begin{array}{ccc}
-\left(k_{1}+k_{2}\right) & 0 & 0 \\
k_{1} & 0 & 0 \\
k_{2} & 0 & 0
\end{array}\right)\right.
$$

Solving the eigenvalue problem of $\underline{\underline{K}}$ :

$$
\underline{\underline{X}}^{-1} \underline{\underline{K}} \underline{\underline{X}}=\left(\begin{array}{ccc}
1 & 0 & 0 \\
k_{1} /\left(k_{1}+k_{2}\right) & 1 & 0 \\
k_{2} /\left(k_{1}+k_{2}\right) & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
-\left(k_{1}+k_{2}\right) & 0 & 0 \\
k_{1} & 0 & 0 \\
k_{2} & 0 & 0
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
-k_{1} /\left(k_{1}+k_{2}\right) & 1 & 0 \\
-k_{2} /\left(k_{1}+k_{2}\right) & 0 & 1
\end{array}\right)=\left(\begin{array}{ccc}
-\left(k_{1}+k_{2}\right) & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right)=\underline{\underline{\Lambda}}
$$

Solving the chemical kinetics problem:

$$
\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=\frac{A_{0}}{k_{1}+k_{2}}\left(\begin{array}{c}
k_{1}+k_{2} \\
-k_{1} \\
-k_{2}
\end{array}\right) \mathrm{e}^{-\left(k_{1}+k_{2}\right) t}+\left(A_{0} \frac{k_{1}}{k_{1}+k_{2}}+B_{0}\right)\left(\begin{array}{l}
0 \\
1 \\
0
\end{array}\right)+\left(A_{0} \frac{k_{2}}{k_{1}+k_{2}}+C_{0}\right)\left(\begin{array}{l}
0 \\
0 \\
1
\end{array}\right)
$$

or component-wise:

$$
A(t)=A_{0} \mathrm{e}^{-\left(k_{1}+k_{2}\right) t} ; \quad B(t)=A_{0} \frac{k_{1}}{k_{1}+k_{2}}\left(1-\mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right)+B_{0} ; \quad C(t)=A_{0} \frac{k_{2}}{k_{1}+k_{2}}\left(1-\mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right)+C_{0}
$$

## Chemical kinetics (some instructive examples, contd.)

II. $\mathrm{B} \stackrel{k_{1}}{\longleftrightarrow} \mathrm{~A} \xrightarrow{\mathrm{k}_{2}} \mathrm{C}$ (contd.)

At every moment in time, the ratio of amounts of compounds $B$ and $C$ generated by the two reactions is constant:

$$
\frac{B(t)-B_{0}}{C(t)-C_{0}}=\frac{k_{1}}{k_{2}}=\lim _{t \rightarrow \infty} \frac{B(t)-B_{0}}{C(t)-C_{0}}
$$

At the end (of time), compound $A$ has completely vanished and only the compounds $B$ and $C$ are present:

$$
\lim _{t \rightarrow \infty}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=\left(0, A_{0} \frac{k_{1}}{k_{1}+k_{2}}+B_{0}, A_{0} \frac{k_{2}}{k_{1}+k_{2}}+C_{0}\right)^{\top}
$$

## Chemical kinetics (some instructive examples, contd.)

III. $A \xrightarrow{k_{1}} B \xrightarrow{k_{2}} C$

$$
\left.\begin{array}{rr}
\dot{A}= & -k_{1} A \\
\dot{B}= & k_{1} A-k_{2} B \\
\dot{C}= & k_{2} B
\end{array}\right\} \equiv\left\{\frac{\mathrm{d}}{\mathrm{~d} t}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=\underline{=}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right), \quad \underline{=}=\left(\begin{array}{ccc}
-k_{1} & 0 & 0 \\
k_{1} & -k_{2} & 0 \\
0 & k_{2} & 0
\end{array}\right)\right.
$$

III. $k_{1} \neq k_{2}$ :

Solving the eigenvalue problem of $\underline{\underline{K}}$ :

$$
\underline{X}^{-1} \underline{\underline{K}} \underline{\underline{X}}=\left(\begin{array}{ccc}
1 & 0 & 0 \\
k_{1} /\left(k_{1}-k_{2}\right) & 1 & 0 \\
1 & 1 & 1
\end{array}\right)\left(\begin{array}{ccc}
-k_{1} & 0 & 0 \\
k_{1} & -k_{2} & 0 \\
0 & k_{2} & 0
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
-k_{1} /\left(k_{1}-k_{2}\right) & 1 & 0 \\
k_{2} /\left(k_{1}-k_{2}\right) & -1 & 1
\end{array}\right)=\left(\begin{array}{ccc}
-k_{1} & 0 & 0 \\
0 & -k_{2} & 0 \\
0 & 0 & 0
\end{array}\right)=\underline{\underline{\Lambda}}
$$

Solving the chemical kinetics problem:

$$
\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=\frac{A_{0}}{k_{1}-k_{2}}\left(\begin{array}{c}
k_{1}-k_{2} \\
-k_{1} \\
k_{2}
\end{array}\right) \mathrm{e}^{-k_{1} t}+\left(A_{0} \frac{k_{1}}{k_{1}-k_{2}}+B_{0}\right)\left(\begin{array}{c}
0 \\
1 \\
-1
\end{array}\right) \mathrm{e}^{-k_{2} t}+\left(A_{0}+B_{0}+C_{0}\right)\left(\begin{array}{l}
0 \\
0 \\
1
\end{array}\right)
$$

or component-wise: $A(t)=A_{0} \mathrm{e}^{-k_{1} t}$;
$B(t)=A_{0} \frac{k_{1}}{k_{1}-k_{2}}\left(\mathrm{e}^{-k_{2} t}-\mathrm{e}^{-k_{1} t}\right)+B_{0} \mathrm{e}^{-k_{2} t} ; \quad C(t)=A_{0}\left(1+\frac{k_{2}}{k_{1}-k_{2}} \mathrm{e}^{-k_{1} t}-\frac{k_{1}}{k_{1}-k_{2}} \mathrm{e}^{-k_{2} t}\right)+B_{0}\left(1-\mathrm{e}^{-k_{2} t}\right)+C_{0}$

## Chemical kinetics (some instructive examples, contd.)

## III. $A \xrightarrow{k_{1}} B \xrightarrow{k_{2}} C$ (contd.)

Assuming $B_{0}=0$, the stationarity condition for $B(\dot{B}=0)$ leads to $B=\frac{k_{1}}{k_{2}} A$. This is fulfilled at time $t_{\hat{B}}=\frac{1}{k_{2}-k_{1}} \ln \left(\frac{k_{2}}{k_{1}}\right)$ (the maximum amount of compound $B$ is available at just this time).
III.b $k_{1}=k_{2}=k$ (set $k_{2}-k_{1}=\varepsilon$, then take limits for $\varepsilon \rightarrow 0$ ):

Solutions for the chemical kinetics problem:

$$
A(t)=A_{0} \mathrm{e}^{-k t} ; \quad B(t)=\left(A_{0} k t+B_{0}\right) \mathrm{e}^{-k t} ; \quad C(t)=A_{0}\left(1-(1+k t) \mathrm{e}^{-k t}\right)+B_{0}\left(1-\mathrm{e}^{-k t}\right)+C_{0}
$$

Assuming again $B_{0}=0$, the stationarity condition for $B$ (see above) now requires that $B=A$. This is fulfilled at time $t_{\hat{B}}=\frac{1}{k}$ (at just this time the maximum amount of compound $B$ is available).
Remark: The case $k_{1}=k_{2}$ requires consideration of a generalized eigenvalue problem for $\underline{\underline{K}}$. This leads to a matrix in Jordan normal form (instead of a diagonal matrix):

$$
\underline{X}^{-1} \underline{K} \underline{=}=\left(\begin{array}{lll}
k & 0 & 0 \\
0 & 1 & 0 \\
1 & 1 & 1
\end{array}\right)\left(\begin{array}{ccc}
-k & 0 & 0 \\
k & -k & 0 \\
0 & k & 0
\end{array}\right)\left(\begin{array}{ccc}
1 / k & 0 & 0 \\
0 & 1 & 0 \\
-1 / k & -1 & 1
\end{array}\right)=\left(\begin{array}{ccc}
-k & 0 & 0 \\
1 & -k & 0 \\
0 & 0 & 0
\end{array}\right)=\underline{\underline{J}}
$$

In all cases, the compounds $A$ and $B$ have completely vanished at the end (of time) and only compound $C$ is present, in total amount $A_{0}+B_{0}+C_{0}$.

## Chemical kinetics (some instructive examples, contd.)

IV. $A \underset{k_{2}}{\stackrel{k_{1}}{\rightleftharpoons}} B$

$$
\left.\begin{array}{rr}
\dot{A}= & -k_{1} A+k_{2} B \\
\dot{B}= & k_{1} A-k_{2} B
\end{array}\right\} \equiv\left\{\frac{\mathrm{d}}{\mathrm{~d} t}\binom{A}{B}=\underline{\underline{\mathrm{K}}}\binom{A}{B}, \quad \underline{\underline{\mathrm{~K}}}=\left(\begin{array}{cc}
-k_{1} & k_{2} \\
k_{1} & -k_{2}
\end{array}\right)\right.
$$

Solving the eigenvalue problem of $\underline{\underline{K}}$ :

$$
\underline{\underline{X}}^{-1} \underline{\underline{K}} \underline{=}=\frac{k_{1}}{k_{1}+k_{2}}\left(\begin{array}{cc}
1 & -k_{2} / k_{1} \\
1 & 1
\end{array}\right)\left(\begin{array}{cc}
-k_{1} & k_{2} \\
k_{1} & -k_{2}
\end{array}\right)\left(\begin{array}{cc}
1 & k_{2} / k_{1} \\
-1 & 1
\end{array}\right)=\left(\begin{array}{cc}
-\left(k_{1}+k_{2}\right) & 0 \\
0 & 0
\end{array}\right)=\underline{\underline{\Lambda}}
$$

Solving the chemical kinetics problem:

$$
\begin{gathered}
\binom{A}{B}=\frac{1}{k_{1}+k_{2}}\left(A_{0} k_{1}-B_{0} k_{2}\right)\binom{1}{-1} \mathrm{e}^{-\left(k_{1}+k_{2}\right) t}+\frac{k_{1}}{k_{1}+k_{2}}\left(A_{0}+B_{0}\right)\binom{k_{2} / k_{1}}{1} \xrightarrow{t \rightarrow \infty} \lim _{t \rightarrow \infty}\binom{A}{B}=\frac{A_{0}+B_{0}}{k_{1}+k_{2}}\binom{k_{2}}{k_{1}} \\
A(t)=\frac{A_{0} k_{2}}{k_{1}+k_{2}}\left(1+\frac{k_{1}}{k_{2}} \mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right)+\frac{B_{0} k_{2}}{k_{1}+k_{2}}\left(1-\mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right) \\
B(t)=\frac{A_{0} k_{1}}{k_{1}+k_{2}}\left(1-\mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right)+\frac{B_{0} k_{1}}{k_{1}+k_{2}}\left(1+\frac{k_{2}}{k_{1}} \mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right)
\end{gathered}
$$

At the end (of time), compounds $A$ and $B$ are both present, not in equal amounts (amount ratio $A / B=1$ ) but in amount ratio

$$
\lim _{t \rightarrow \infty} \frac{A(t)}{B(t)}=\frac{k_{2}}{k_{1}}
$$

## Chemical kinetics (some instructive examples, contd.)

$\mathbf{V} . \ldots \xrightarrow{\mathrm{k}_{3}} A \xrightarrow{\mathrm{k}_{1}} \mathrm{~B} \xrightarrow{\mathrm{k}_{2}} C \xrightarrow{\mathrm{k}_{3}} A \xrightarrow{\mathrm{k}_{1}} \ldots$

$$
\left.\begin{array}{cc}
\dot{A}= & -k_{1} A+k_{3} C \\
\dot{B}= & k_{1} A-k_{2} B \\
\dot{C}= & k_{2} B-k_{3} C
\end{array}\right\} \equiv\left\{\frac{\mathrm{d}}{\mathrm{~d} t}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=\underline{=}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right), \quad \underline{=}=\left(\begin{array}{ccc}
-k_{1} & 0 & k_{3} \\
k_{1} & -k_{2} & 0 \\
0 & k_{2} & -k_{3}
\end{array}\right)\right.
$$

Solving the eigenvalue problem of $\underline{\underline{K}}$ :

$$
\begin{gathered}
0=\operatorname{det}(\underline{\underline{K}}-\lambda \underline{\underline{E}})=-\lambda\left(\lambda^{2}+\left(k_{1}+k_{2}+k_{3}\right) \lambda+\left(k_{1} k_{2}+k_{2} k_{3}+k_{3} k_{1}\right)\right) \\
\lambda_{1,2}=-\frac{1}{2}\left(k_{1}+k_{2}+k_{3}\right) \pm \frac{1}{2} \sqrt{D} ; \quad \lambda_{3}=0
\end{gathered}
$$

with discriminant $D=k_{1}^{2}+k_{2}^{2}+k_{3}^{2}-2\left(k_{1} k_{2}+k_{2} k_{3}+k_{3} k_{1}\right)$.
(i) $D=0\left(\lambda_{1}=\lambda_{2} \in \mathbb{R}\right)$ describes a cone in the $1^{\text {st }}$ octant $\left(k_{1}>0, k_{2}>0, k_{3}>0\right)$ with apex at the origin and rotation axis along the space diagonal $\left(k_{1}, k_{2}, k_{3}\right)^{\top}=\alpha(1,1,1)^{\top}$, which touches the coordinate system planes ( $k_{i}=0$ ) along their angle-bisecting diagonal lines ( $k_{j}=k_{l}, j \neq i \neq I$ ).
(ii) $D<0\left(\lambda_{1}=\lambda_{2}^{*} \in \mathbb{C}\right)$ inside that cone.
(iii) $D>0\left(\lambda_{1} \neq \lambda_{2} \in \mathbb{R}\right)$ in the remaining parts of the 1st octant outside the cone.

If $\lambda_{1} \neq \lambda_{2}$ :

$$
\begin{gathered}
\underline{\underline{X}}=\left(\underline{x}_{1}, \underline{x}_{2}, \underline{x_{3}}\right)=\left(\begin{array}{ccc}
\left(\lambda_{2}+k_{1}\right) / k_{2} & \left(\lambda_{1}+k_{1}\right) / k_{2} & k_{3} / k_{1} \\
\left(\lambda_{2}+k_{3}\right) / k_{2} & \left(\lambda_{2}+k_{3}\right) / k_{2} & k_{3} / k_{2} \\
1 & 1 & 1
\end{array}\right), \quad \underline{\underline{\Lambda}}=\operatorname{diag}\left(\lambda_{1}, \lambda_{2}, 0\right) \\
\operatorname{det}(\underline{\underline{X}})=\frac{\lambda_{2}-\lambda_{1}}{k_{1} k_{2}^{2}}\left(k_{1}^{2}+\left(\lambda_{1}+\lambda_{2}\right) k_{1}-k_{2} k_{3}\right)=\frac{\sqrt{D}}{k_{2}}\left(\frac{k_{3}}{k_{1}}+\frac{k_{3}}{k_{2}}+1\right)
\end{gathered}
$$

## Chemical kinetics (some instructive examples, contd.)

## $V . \ldots \xrightarrow{k_{3}} A \xrightarrow{k_{1}} B \xrightarrow{k_{2}} C \xrightarrow{k_{3}} A \xrightarrow{k_{1}} \ldots$ (contd.)

Form of solutions for the chemical kinetics problem ( $\lambda_{1} \neq \lambda_{2}$, assuming $\left.\operatorname{Re}\left(\lambda_{i}\right)<0\right)$ :

$$
\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=c_{1} \underline{x}_{1} \mathrm{e}^{\lambda_{1} t}+c_{2} \underline{x}_{2} \mathrm{e}^{\lambda_{2} t}+c_{3}\left(\begin{array}{c}
k_{3} / k_{1} \\
k_{3} / k_{2} \\
1
\end{array}\right) \in \mathbb{R}^{3}
$$

This contains exponentially decaying parts for $D \geq 0\left(\lambda_{1}, \lambda_{2}\right.$ both real)
and exponentially damped oscillating parts for $D<0\left(\lambda_{1}=\lambda_{2}^{*} \in \mathbb{C}\right)$.
At the end (of time), the amounts of compounds A, B and C are

$$
\lim _{t \rightarrow \infty}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=c_{3}\left(\begin{array}{c}
k_{3} / k_{1} \\
k_{3} / k_{2} \\
1
\end{array}\right)=\frac{c_{3}}{k_{1} k_{2}}\left(\begin{array}{l}
k_{2} k_{3} \\
k_{3} k_{1} \\
k_{1} k_{2}
\end{array}\right)
$$

Equal amounts of compounds $\mathrm{A}, \mathrm{B}$ and C are expected (or, more important perhaps, cannot be avoided) iff (if and only if) all rate constants $k_{i}$ are equal:

$$
\lim _{t \rightarrow \infty}\left(\begin{array}{l}
A \\
B \\
C
\end{array}\right)=c_{3}\left(\begin{array}{l}
1 \\
1 \\
1
\end{array}\right) \quad\left(k_{1}=k_{2}=k_{3}\right)
$$

logistic growth
$\dot{Q}=r Q-\operatorname{alpha} Q^{2}$
PF Verhulst (1804-1849)

## On Responsibility




```
FHBLEMRE UND EUGENIK.
```





```
POLITR DES NS-SVAATES ALS AUSBLLDEF VON SSSARZIEN UND GHES
```



```
WEISE UND ZWANESSTERILTSATIONEN LESIGIEN SIE EINEN ARTIVEN
BETTMAGZZL SELERTION UND NVORD,
DIE VOM REICHSFORSCHUNGSAAT BEWILLIGIEN UND VON DEF DEUISCAEN
FORSCHUNGSLCENEINSCHANF FINANZZERTEN, ZWILLINESFORSCHINGEN DES
SCHULERS UND PERSOZNRICHEN NITARBETTERS VON VERSCHUEF
DOSEF NENGELE TM KZ, AUUSCHWOITZ WURDEN IN DIESEM GEBAUDE GEPLANY
UND DURCH UNTERSUCRUNEEN ANY ORGANEN SELEKTIERTERUND ERIVOFIDEIER
W\mp@code{FILNEE UNTERSTUUZZ5}
```



```
FÖR GENETLK BIS 1965 IN NÜNSTEF.
MISSENSCHAFTLERMMABEN GNMALTYUND FOLGEN IMRER WISSENSCHAFTLICHENS
AFBET zv, verantworiten.
```


## English translation of the last sentence:

"Scientists are responsible for the content and consequences of their scientific work."

Memorial plaque at the building of the Otto Suhr Institute for Political Science, FU Berlin, Ihnestr. 22, Berlin-Dahlem (former Kaiser Wilhelm Institute of Anthropology, Human Heredity and Eugenics, 1927-1945), reminding us of the institute and its role during the time of National Socialism Picture (taken in 2005): https://de.wikipedia.org/wiki/Otto-Suhr-Institut (CC BY-SA 3.0, accessed 17 Apr 2024)

## From the preface of Anastas \& Warner: Green Chemistry

## Some topic-related thoughts from Berlin street art

"Nature does not need us / But we need Nature"


A group of artists with B Wagin: Weltbaum II - Werden, Sein, Vergehen, 1985/1986 Street art mural, S-Bahn Savignyplatz, Berlin, pictures taken 2022 (left) and 2021 (right)

Street art mural created at the occasion of the 750-years anniversary of Berlin, 1987 - B Wagin (1930-2021),
https://en.wikipedia.org/wiki/Ben_Wagin Landesdenkmalamt Berlin, 08.12.2022, (C)Thorsten Dame (Link) Berliner Zeitung, 09.06.2021, (C)Berliner Zeitung / Jens Blankennagel (Link)

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"Many small people doing many small things in many small places can change the face of the world."

- African wisdom


K Alavi \& M Raoux (without title), 1990 / East Side Gallery, Berlin
Picture: https://es.wikipedia.org/wiki/East_Side_Gallery (CC BY-SA 3.0, accessed 17 Apr 2024) see also: Google Arts \& Culture, East Side Gallery (Link)

Demonstration(s) with objects from everyday life (if time permits)

## Take-home messages

- We can create/destroy neither matter nor energy (conservation of mass/energy)
- Only three types of thermodynamic systems exist (isolated, closed, open)
- Flow of matter/energy is unavoidably linked to "losses" (entropy generation). Only in isolated systems, this ends when entropy has reached its maximum
- While matter/energy flow through open systems never stops producing entropy, it does (or can) create "structure" (patterns in space and/or time, life)
- When our usage of things/goods/stuff has come to an end: Which reusability quality has that stuff when it is released by us? Reusability quality for whom, for which (possibly other) forms of life on this planet?
- What about chemical elements (and their compounds) that play no (known/essential) role for life, like B or F? Prohibition?
- Enjoy life, every single day, and don't stop thinking for yourself
"Sustainability is good, if everyone contributes to it."
(from a recent advertisement campaign of a German drugstore chain)



Farewell, dear masters ( $\mathrm{f} / \mathrm{m} / \mathrm{d}$ ) of the 80 elements!*

Thank you
for your attention!
Questions? Welcome anytime!


[^0]:    P Harris (Baloo) \& B Reitherman (Mowgli): The Bare Necessities, The Jungle Book, Disney, 1967

