Explaining shifts in the unemployment rate with productivity slowdowns and accelerations: a co-breaking approach*

by Sven Schreiber†

This version: January 2009

Abstract

We investigate the controversial issue whether unemployment is related to productivity growth in the long run, using U.S. data in a framework of infrequent mean shifts. Tests find (endogenously dated) shifts around 1974, 1986, and 1996, system techniques indicate that the shifts are common features, and the implied long-run link between the two variables is negative. Therefore the secular decline of unemployment since the mid 1990s indeed stemmed from higher average productivity growth. The initial and final regimes are essentially equal, thus supporting theories that explain the productivity slowdown by a slow adoption process of IT with associated learning costs.

Keywords: productivity slowdown, growth, NAIRU level, common shifts

JEL codes: E24, C32, O40

*I thank Michael Massmann and participants of the IEA, EALE, and ESEM meetings, and of seminars in Berlin, Frankfurt, and Hamburg for helpful comments. Thanks also go to Alexandra Spitz for useful references.

†Macroeconomic policy institute (IMK) at Hans Böckler Foundation, and Goethe University Frankfurt. Address: Hans-Böckler-Stiftung, Hans-Böckler-Str. 39, D-40476 Düsseldorf, Germany. Fax: +49(0)211-7778-4332. Email: svetosch@gmx.net

1 Introduction

Whether there exists a long-run connection between unemployment and productivity is controversial among economists. It is clear that the *level* of (trending) productivity cannot reasonably have a long-run influence on the (non-trending) unemployment rate. However, a much stronger hypothesis is that "neither the level or rate of change of productivity has any long-run effect on the unemployment rate" (Stiglitz, 1997, p. 7; see for example Ball and Mankiw, 2002, for similar statements). Such a claim is in conflict with several model classes, including standard search and matching models of unemployment,[1] but also models where wage setting depends on reservation wages which are partly backward-looking (for whatever reason, see below). In this paper we show that changes in productivity growth indeed have a prolonged impact on unemployment, but that in the 1990s the USA have returned to a regime of "normal" values of average growth and unemployment.

The contributions of this paper can be summarized in three main points: One consists of a simple model in which we highlight that the dependence of the reservation wage on lagged (real) wages is sufficient to explain the observed negative long-run link between unemployment and productivity growth.[2] This is an attractive result because in reality unemployment benefits are partially indexed to past wages, and therefore assuming such a link to past wages is completely natural.[3] Another point is that we provide further evidence on the existence of a long-run connection between productivity growth and unemployment. Using new co-breaking methods we confirm the findings in Pissarides and Vallanti (2007) or Tripier (2006), in contrast to the results of Muscatelli and Tirelli (2001) which were insignificant for the USA. And finally, in our bivariate system we can identify separate regimes and we show that the post-1996 equilibrium is essentially equal to the one of the pre-1974 period. In that sense the productivity slowdown and the associated elevated average unemployment rates are history. These findings support the view that the

[1] The usual references are Aghion and Howitt (1994) and Mortensen and Pissarides (1998); see Prat (2007) for a recent interesting extension.

[2] Assuming such a backward-looking reservation wage goes back to Blanchard and Katz (1999). See Tripier (2006) for a richer model embedding the same feature.

[3] Hogan (2004) showed that such a link is also significant in British micro data.

productivity slowdown was a prolonged but ultimately transitory phenomenon, induced by the slow diffusion of information technology (IT) with associated temporary adoption costs.

With respect to empirical methods, we start by using the univariate techniques developed in Bai and Perron (1998) and Bai and Perron (2003) to estimate the number and the dates of the breaks. However, the main empirical novelty of this paper is the system analysis to test whether the shifts identify a common structure. This approach uses the concept of "co-breaking" (see Hendry and Massmann, 2007, and also Krolzig and Toro, 2002) which is a special case of the notion of common features (Engle and Kozicki, 1993).[4] The number of variables in a co-breaking analysis is conceptually limited by the number of shifts, which explains why there have not been many economic applications of co-breaking methods until now. However, since longer time series as well as methods to estimate shifts are becoming more widely available, the co-breaking approach promises to be a very useful tool for econometric modellers.

The remainder of this paper is structured as follows: In section 2 we present a stylized and stripped-down theoretical model to highlight the long-run implications of backward-looking reservation wages. Afterwards 3 contains a brief description of the univariate mean-shift tests and their results. Then in section 4 we formulate the statistical framework for the system analysis, explain the test and estimation procedures, and report the system results. Finally, section 5 provides some conclusions.

2 A stripped-down model of steady-state unemployment

In this section we present a bare-bones theoretical example in which the steady-state level of the labor market tightness depends on the growth rate of productivity, assuming reservation wages that partially depend on past real wages. The analysis focuses on the labor

[4] Another possible recent method would be the approach described by Qu and Perron (2007). However, our method of estimating the break dates separately for each series enables us to use a series with a different frequency like monthly unemployment. It also renders it easy to check whether the estimated dates indeed coincide across series.

market and is partial equilibrium in the sense that technical progress as well as human and physical capital accumulation are not endogenized. It is related to models such as the one in Tripier (2006). It is also clear that partially backward-looking reservation wages are *not necessary* for a long-run link between productivity growth and unemployment, see Prat (2007) and the references therein (including standard search-matching models). However, an important institutional reason for incorporating the dependence of reservation wages on its own past into a model is that in all states of the USA unemployment benefits are tied to past wages (albeit to varying extents), see U.S.-Government (2004, p. 12). Since unemployment benefits influence the cost of search in matching models, and the outside option in bargaining models, there is a direct link to reservation wages.

The unemployment rate is of course not a perfect indicator for labor market tightness. However, other conceivable measures also suffer from various weaknesses: For example, the well-known „Help-wanted advertising index" is subject to various structural changes as described in Valletta (2005). Most notably the rise of the internet makes reliance on newspaper ads to measure trends in labor market tightness in recent years seem awkward. The Bureau of Labor Statistics' "job openings and labor turnover survey" (JOLTS) is only available from 2000 onwards. Also, while observed unemployment rates are in principle also influenced by changes of labor force participation rates, a look at the data reveals that this cannot explain the pronounced shifts that we find (see e.g. Bradbury, 2005).

2.1 Production and labor demand

We assume a "right-to-manage" framework which means that firms can unilaterally determine how much labor they employ at the going wage rate (as opposed to bargaining over wages and employment levels simultaneously). This implies that the labor market outcome will always lie on the labor demand curve. This labor demand is derived from a constant-returns-to-scale production function of the standard constant-elasticity-

of-substitution (CES) form:

$$Y = \gamma\Big(\delta L_E^{(\sigma-1)/\sigma} + (1-\delta)K^{(\sigma-1)/\sigma}\Big)^{\sigma/(\sigma-1)}, \tag{1}$$

where Y is aggregate output, L_E are effective labor inputs, and K is capital. Parameters are denoted by Greek letters; $\gamma > 0$ is the scale parameter, $\delta \in (0;1)$ is the distribution parameter, and $\sigma > 0$ is the elasticity of factor substitution; for $\sigma = 1$ we obtain the special case of the Cobb-Douglas function. Allowing for imperfect competition both in factor and output markets is straightforward as long as the respective mark-ups are assumed constant or at least stationary.

We adopt the interpretation from Durlauf and Quah (1999) that labor is enhanced with embodied human capital H and (disembodied) Harrod-neutral technological progress A, which are subsumed under $E \equiv AH$, such that E is a joint enhancement term. Note that the growth of E may slow down (or speed up) even when dA/dt is constant, depending on variations in the accumulation or depreciation speed of human capital. This distinction may become relevant for example in the story of Greenwood and Yorukoglu (1997). Altogether, effective labor inputs (L_E) therefore depend on hours worked (L) and the described enhancement factor:

$$L_E = EL \tag{2}$$

Standard derivations yield the following implicitly defined labor demand equation (conditional on product demand Y):

$$q_t = c_{ld} + \sigma w_t + (1-\sigma)e_t + \varepsilon_{ld,t}, \tag{3}$$

where lower-case latin letters denote logarithms and we have attached time indexes to the variables; $q_t \equiv y_t - l_t$ is observed hourly labor productivity, and w_t are real hourly labor costs. We will often refer to w_t simply as "wages". The constant c_{ld} contains various parameters (elasticity and mark-up terms, for example) and is not directly important here.

Finally, $\varepsilon_{ld,t}$ is a stationary error term with unconditional mean zero.

2.2 Wage setting

A convenient summary of a wage setting relation is given in Blanchard and Katz (1999):

$$w_t = \mu r_t + (1-\mu)q_t - \beta_u u_t + \varepsilon_{w,t},\ 0 \leq \mu \leq 1, \beta_u > 0 \tag{4}$$

This states that real wages w_t are a function of the reservation wage r_t (including unemployment benefits etc.), the observed labor productivity q_t, and the (inverse) tightness of the labor market u_t. As discussed by Blanchard and Katz (1999) this formulation is compatible with bargaining models (including insider-outsider models) as well as with models that incorporate matching frictions, or efficiency wage considerations. Note that this wage function is linearly homogeneous in the reservation wage and in productivity, such that wages and productivity may grow at the same rate in the long run even with $\mu \neq 0$.

We follow Blanchard and Katz (1999) one step further and also posit that the reservation wage depends on past real wages and on current productivity levels in a linear-homogeneous fashion:

$$r_t = c_r + \lambda w_{t-1} + (1-\lambda)q_t,\ 0 < \lambda \leq 1 \tag{5}$$

Equation (5) is not only an intuitively plausible assumption due to the mentioned institutional characteristics of unemployment benefits, but is also supported by recent micro evidence: Hogan (2004) finds that the elasticity of the reservation wage with respect to past wages is significantly positive in household panel data for the UK, with a point estimate of about 0.3. Inserting (5) into (4) yields:

$$\Delta w_t = \mu c_r - (1-\mu\lambda)(w_{t-1} - q_{t-1}) - \beta_u u_{t-1} - \beta_u \Delta u_t + (1-\mu\lambda)\Delta q_t + \varepsilon_{w,t} \tag{6}$$

So we see that the (lagged) labor share $(w-q)_{t-1}$ in general enters the wage setting curve, which is the major difference with respect to an expectations-augmented (wage) Phillips curve.[5]

2.3 Equilibrium

Combining the standard wage setting curve (6) with the labor demand function (3) yields:

$$\Delta w_t = c_{eq} - \frac{\alpha-\mu\lambda}{\alpha}(w_{t-1}-e_{t-1}) - \frac{\beta_u}{\alpha}u_t + \frac{\alpha-\mu\lambda}{\alpha}\Delta e_t + \varepsilon_{eq,t} \tag{7}$$

with $\alpha \equiv 1-\sigma(1-\mu\lambda)$, $c_{eq} \equiv (\mu c_b + (1-\mu\lambda)c_{ld})/\alpha$ and $\varepsilon_{eq,t} \equiv (\varepsilon_{w,t} + (1-\mu\lambda)\varepsilon_{ld,t})/\alpha$. The term in square brackets is another equilibrium correction term, now written in terms of the unobservable labor-enhancing variable e_t instead of observable productivity.[6] The sign of $\beta_u/(\alpha-\mu\lambda)$ and thus whether unemployment co-moves with adjusted wages $w_t - e_t$ depends on the substitution elasticity. For example, if labor and capital are gross complements ($\sigma < 1 \Rightarrow \beta_u/(\alpha-\mu\lambda) > 0$), in equilibrium higher unemployment levels would be accompanied by *lower* wages relative to technology and human capital accumulation.

It is also worth acknowledging that this model does not automatically exhibit system stability under exogenous technical progress. The relevant condition on the adjustment coefficient, $-(\alpha-\mu\lambda)/\alpha < 0$, depends on the parameter values, i.e. the system is unstable *iff* $1 < \sigma \leq 1/(1-\mu\lambda)$.[7] However, given that most studies find that σ is quite a bit smaller than unity (possibly around 0.5), this case does not appear to be too relevant, apart from the fact that endogenizing the labor-enhancing variables e_t –which is perfectly

[5]However, from an econometric point of view it is unfortunate that Blanchard and Katz (1999) and OECD (1997) call this an "error correction term" because this implicitly assumes that the labor share is stationary, which is clearly not the case in many countries. In the general case, covering non-stationary labor shares as well as unemployment rates, the actual error correction term is given by $w_{t-1} - q_{t-1} + \beta_u/(1-\mu\lambda)u_{t-1}$.

[6]In the knife-edge case of Cobb-Douglas production ($\sigma = 1$) this term would reduce to $-(1/\mu\lambda)[\beta_u u_{t-1}]$.

[7]The same coefficient and thus the same condition appears if instead of a wage equation (with Δw_t on the left-hand side) the equation for productivity (with Δq_t on the left-hand side) is used.

compatible with this model, where e_t is simply left unmodelled– could solve this potential instability (see e.g. Acemoglu, 2003).

We will assume *balanced growth* regimes in the following sense: the equilibrium growth rates of wages and productivity are assumed to be the same, $\Delta\overline{w} = \Delta\overline{q} = g_{(t)} > 0$, where we have attached a time index in parentheses to reflect the assumption that this steady-state growth rate may change, but only occasionally.[8] This implies that the (log) labor share is not drifting. We saw before that this also implies $\Delta\overline{e} = g_{(t)}$. Unemployment itself will then on average be constant in each regime, therefore $\Delta\overline{u} = 0$. The adjusted wage level $w - e$ is not drifting, but its status is that of an initial condition and we write it accordingly as $(w-e)_0$. Then the balanced growth equilibrium is given by:

$$\overline{u}_{(t)} = \frac{1}{\beta_u}\left(c_{eq}\alpha - \mu\lambda g_{(t)} - (\alpha - \mu\lambda)(w-e)_0\right) \tag{8}$$

Thus there are many parameters that determine steady-state unemployment in general, including the intercept of the reservation wage equation (c_r, which is in c_{eq}), product and labor market competitive environments (the respective elasticities are in c_{ld}, which in turn is in c_{eq} as well), and of course the parameters of wage setting and factor substitution.

However, the important additional factor is the balanced growth rate $g_{(t)}$. Except in the extreme case $\mu\lambda = 0$ the equilibrium growth rate of productivity has a negative influence on steady-state unemployment. The cause of this effect is sometimes interpreted as slowly adjusting wage aspirations, because $\lambda \neq 0$ means that wages are partially determined by past wages, instead of being fully determined by contemporaneous productivity.

3 Mean shifts in productivity growth and unemployment

We first apply the univariate methods developed by Bai and Perron (1998, 2003) that are capable of estimating and testing for the number of breaks and their location in the sample. This approach was applied by Hansen (2001) to labor productivity growth of the U.S.

[8] In the empirical analysis we identify two such shifts over a span of roughly fifty years.

manufacturing sector, and by Papell et al. (2000) to annual data of the unemployment rates of several OECD countries up to 1997. A recent example of the increasingly common practice of accounting for breaks in productivity growth is Fernald (2007), and an earlier (less formal) finding of a break in U.S. unemployment in 1974 is given in Evans (1989). Because unemployment rates do not refer just to manufacturing, we accordingly use productivity growth of the entire business sector (quarterly, denoted by Δq_t), and compared to Papell et al. (2000) and Evans (1989) we use longer samples and higher-frequency data for unemployment rates, namely monthly series (u_t^m).[9]

The univariate framework for the multiple-shifts test is given by

$$x_t = \delta_j + \varepsilon_{uni,t}, \quad t = T_{j-1}+1, \ldots, T_j, \quad T_0 = 0,\ T_{m+1} = T, \tag{9}$$

for $j = 1, \ldots, m+1$, where m is the number of shifts, so there are $m+1$ regimes. In each regime there is a potentially different intercept δ_j, but the break points T_j are unknown. It is important to note that the residual $\varepsilon_{uni,t}$ need not be white noise nor homogeneous; in the Gauss program that is provided by Perron on the website of the Journal of Applied Econometrics and that was used for these tests the covariance estimate accounts for that. We always allow for as much heterogeneity as possible. There are two differences between the applications to the two variables: We use the pre-whitening option for the residual covariance estimation for Δq_t but not for u_t^m, because with the near-unit-root serial correlation in u_t^m the pre-whitening procedure seemed to break down, producing erratic and unreliable test results. Also, we use a trimming value of 0.15 for Δq_t but 0.1 for u_t^m, because of the different sample sizes ($T = 238$ for Δq_t, $T = 716$ for u_t^m). The trimming value determines the minimal length of potential regimes; according to Bai and Perron (2003), when heterogeneity in the residuals is allowed but no serial correlation, a trimming of 0.15 is appropriate for $T = 120$, implying a minimal regime length of 18

[9]Productivity growth Δq_t is measured as the first time difference of quarterly log hourly real labor productivity of the business sector (seasonally adjusted), the source is the Bureau of Labor Statistics (BLS), sample 1948q1-2007q2. Unemployment is a survey-based (16 years and over) seasonally adjusted measure in percentage points also provided by the BLS, sample 1948m1-2007m8.

observations. Our choices mean minimal regime lengths of 36 (for Δq_t) or 72 (for u_t^m) observations while also allowing for serial correlation.

The description of the global minimization algorithm for the break date estimation in Bai and Perron (2003) will not be reproduced here, as we essentially use it as a black box. However, it is necessary to introduce the various test hypotheses and respective statistics. All their distributions are non-standard.

First there are supremum-type test statistics for the fixed null hypothesis $H_0 : m = 0$ against various fixed alternatives: $H_{1,j} : m = j$ for $j = 1, ..., m_{max}$, where we set $m_{max} = 5$. Obviously, the power of each of these tests will depend on the true number of breaks. Also they are not independently distributed, but nominal significance of at least one of these tests may point to the existence of at least one break. Let us denote these statistics by $supF(j|0)$. Next, $UDmax$ and $WDmax$ are two related statistics that do not specify the precise number of breaks under the alternative, thus $H_0 : m = 0$ and $H_1 : 1 < m \leq m_{max}$. They differ in their weighting schemes.

Finally, and most importantly, Bai and Perron (2003) suggest a testing sequence that in effect estimates the number of breaks. The most restricted model of course imposes $m = 0$ which is the starting point. Then the procedure partitions the sample according to all previously found breaks and then tests for single additional breaks in each subsample. The procedure stops at the first non-significant test. For $j = 1, ..., m_{max}$ the nested sequence of hypothesis pairs is then $H_{0,j} : m = j - 1$ against $H_{1,j} : m = j$. Let us call these statistics $supF(j|j-1)$.

Another possible approach would be to use information criteria for model selection to determine the number of breaks. However, they severely overestimate the number of breaks with serially correlated errors and do not allow for residual heterogeneity across regimes and are therefore not attractive.

In table 1 we report the test results displaying only the significance level, not the actual numbers and critical values which would be distracting and not really helpful. The tests clearly indicate that there are some mean shifts in the variables over the analyzed samples.

Table 1: Tests for number of mean shifts in the variables

	$UDmax$	$WDmax$	$supF(1\|0)$	$supF(2\|0)$	$supF(3\|0)$
Δq_t	*	*	*	**	*
u_t^m	***	***	*	***	***

	$supF(2\|1)$	$supF(3\|2)$	$supF(4\|3)$
Δq_t	**	n.s.	-
u_t^m	***	***	n.s.

	sample	T	trimming
Δq_t	1948q1-2007q2	238	0.15
u_t^m	1948m1-2007m8	716	0.10

Notes: Upper panel tests for existence of at least one break, lower panel has sequence of tests to determine the number. Significance denoted by * (10%), ** (5%), *** (1%), or "n.s." (p-value greater than 10%). To avoid clutter, $supF(4|0)$ and $supF(5|0)$ are not reported. "q" and "m" stand for quarter and month, respectively. For the meaning of the trimming value see the text.

For productivity growth Δq_t the sequence clearly indicates two breaks, while for u_t^m even three breaks are found. We will revisit the issue of the "extra" break in unemployment in the system context below, but at this point we already conjecture that the smaller number of observations for the productivity growth series due to the lower available frequency in combination with its high short- and medium-run volatility might conceal a third break.

The resulting break date and intercept estimates for the various regimes are collected in table 2. Most importantly, at the end of the 1990s the means of both variables essentially return to their original pre-1970s values. Secondly, the dates of the first and last breaks roughly coincide across the two variables after accounting for estimation uncertainty, which is a requirement for common shifts. However, while the break dates are estimated quite precisely for the unemployment rate, there is considerable uncertainty for productivity growth. It is also interesting that with this economy-wide data we are able to formally confirm the stylized fact of the productivity slowdown after 1973, whereas Hansen (2001) only found a single break in the 1990s (for manufacturing productivity

Table 2: Estimated means and their break dates

	productivity growth, Δq_t	unemployment rate, u_t^m
first regime	3.19	4.8
	(0.38)	(0.21)
	65q4←1973q2→87q2	74m5←1974m11→76m8
intermediate regime	1.42	7.5
	(0.32)	(0.15)
	-	86m12←1986m12→89m1
intermediate regime 2	-	6.1
		(0.08)
	91q4←1995q4→end	95m7←1996m6→96m11
last regime	2.97	4.8
	(0.33)	(0.10)

Notes: "q" and "m" stand for quarter and month, respectively. Numbers in parentheses are standard errors. The unemployment rate is given in percentage levels, and the productivity growth in annualized percentage rates. The estimates and 95% confidence intervals for the break dates ("←→") were calculated with the Gauss program by Perron mentioned in the text.

data) or additional breaks in 1964 and 1982 (for the industrial production index divided by hours worked). Interestingly, Hansen's additional break in the 1980s would roughly match the extra third break in unemployment. With respect to the unemployment rate we find the same breaks as was possible for Evans (1989) and Papell et al. (2000) with their respective samples, but find an additional break in 1996.

4 System analysis of common shifts

Since the used variables are observed at different frequencies (quarterly vs. monthly), we first converted the monthly unemployment rate to quarterly data by averaging the respective observations, and we denote the quarterly series simply by u_t.[10]

[10]The empirical analysis was performed with PcGive (Doornik and Hendry, 2001) and scripts written by the author in the (numerical) Python language for scientific computing (see www.scipy.org).

4.1 The statistical framework

To analyze the relation between the breaks in the variables we use a vector-autoregressive (VAR) model augmented with shift dummies. Although there are some important differences it will be helpful to keep in mind the analogy to a standard cointegrated VAR, see Johansen (1995). The common shifts in our model replace the common stochastic trends driving the $I(1)$ variables in a cointegration model. The model we use is closely related to the ones discussed in Hendry and Massmann (2007) and is given by:

$$\mathbf{x_t} = \mathbf{c_0} + \delta\gamma'\mathbf{s_t} + \mathbf{e_t}, \quad \mathbf{e_t} = \sum_{i=1}^{g} \Gamma_i L^i \mathbf{e_t} + \varepsilon_{\mathbf{multi,t}}, \quad t = 1, ..., T \tag{10}$$

Here $\mathbf{x_t}$ is the $(n \times 1)$-variable vector, where in our case $\mathbf{x_t} = (\Delta q_t, u_t)'$, g is the lag length, L is the lag operator, the Γ_i are coefficient matrices, and $\mathbf{c_0}$ is a constant term and thus the intercept of the first regime before any breaks happen. As in the previous section, m is still the number of (common) shifts, and the break points are still denoted by T_j $(j = 1, ..., m)$. Thus $\mathbf{s_t} = (1(T_1 < t \leq T_2), ..., 1(T_m < t \leq T))'$ is the $(m \times 1)$-vector of additional intercepts for regimes $2, ..., m+1$, using the standard indicator function $1(.)$ that takes the value of one if its argument is true, else zero. The common shifts are assumed to be the only source of non-stationarity, so we impose that the lag polynomial roots $I - \sum_{i=1}^{g} \Gamma_i z^i$ are all stable.

The coefficient matrix in front of $\mathbf{s_t}$ has reduced rank $b < n$ if co-breaking exists (i.e., if the breaks are linearly related) and thus can be factored into the two matrices δ and γ of dimensions $n \times b$ and $m \times b$ respectively, and full rank b each. As is well known from cointegration analysis, this decomposition is only unique after imposing normalizing and identifying restrictions. The matrix γ determines the proportions of the various shift dummies in each "shift package", while δ loads the different shift packages into the variables.

The number $n - b$ describes the number of linearly independent stationary combinations of the variables and is called the "co-breaking rank". We note that if $n > m$, i.e. if

we have more breaking variables than breaks, then there will always exist a linear combination of variables without shifts. This is usually called "spurious co-breaking" and does not allow any meaningful interpretation.[11] Thus we will restrict ourselves to the situation $n \leq m$, where it is clear that we have $b \leq n$. But of course if $b = n$ the shifts in the variables are linearly independent and thus no co-breaking occurs.

We denote by $\beta_\perp$ any orthogonal complement of a given $(n \times b)$-matrix β with full column rank.[12] It then follows from (10) that pre-multiplying $\mathbf{x_t}$with $\delta'_\perp$ yields the co-breaking relation

$$\delta'_\perp \mathbf{x_t} = \delta'_\perp \mathbf{c_0} + \delta'_\perp \mathbf{e_t}. \tag{11}$$

Therefore we call $\delta'_\perp$ the co-breaking matrix and $\delta'_\perp \mathbf{x_t}$ is stationary.

In order to estimate the various parameters of (10), Hendry and Massmann (2007) take the natural approach of specifying a slightly generalized VAR system which removes the transitional observations after each break with the help of impulse dummies (of the form $\{...,0,0,1,0,0,0,...\}$). They call the resulting system an unrestricted "UVAR" because the coefficients of the impulse dummies do not reflect the complicated nonlinear restrictions that would be needed to be strictly equivalent to the representation in (10). Such unrestricted impulse dummies are often used in the structural break literature, see for example Johansen et al. (2000); Saikkonen and Lütkepohl (2000). This UVAR is given by:

$$\mathbf{x_t} = \sum_{i=1}^{g} \Gamma_i L^i \mathbf{x_t} + \mathbf{k_0} + \alpha\gamma' \mathbf{s_t} + impulses + \varepsilon_t \tag{12}$$

The interesting parameters of the previous representation are related to the ones in the UVAR via the relations $\mathbf{c_0} = \Gamma(1)^{-1}\mathbf{k_0}, \quad \delta = \Gamma(1)^{-1}\alpha$.

It is therefore essential for the analysis of common shifts to determine the rank of the shift coefficient matrix $\alpha\gamma'$. Given that no integrated variables are involved this is a

[11] However, the term "spurious" is perhaps a little too strong, because the fact that the break dates of the variables coincide already contains valuable information and is by no means automatic.

[12] The matrix $\beta_\perp$ has dimension $n \times n - b$, full column-rank, is a basis of the null space of β, and satisfies $\beta'_\perp \beta = 0$.

standard reduced rank regression problem. We write the system in matrix form as

$$X = Y\Gamma + S\gamma\alpha' + E_{multi}, \tag{13}$$

where X is $T \times n$, the $(T \times gn + 1 + \text{numbers of impulse dummies})$-matrix Y collects the observations on lags, the constant, and the impulse dummies, Γ is the corresponding coefficient matrix, S is the $T \times m$ data matrix of the shift dummies. As in the Johansen procedure, the unrestricted terms in A are first concentrated out:

$$R_0 = (I_T - Y(Y'Y)^{-1}Y')X, \qquad R_1 = (I_T - Y(Y'Y)^{-1}Y')S \tag{14}$$

Then the moment matrices are formed:

$$M_{ij} = T^{-1}R_i'R_j \quad i,j = 0,1 \tag{15}$$

And we have $M_{01} = M_{10}'$. Then we solve the generalized eigenvalue problem corresponding to:

$$\lambda_i M_{11} v_i = M_{10} M_{00}^{-1} M_{01} v_i, \tag{16}$$

where λ_i are the eigenvalues with respective eigenvectors v_i. The number of (significant) non-zero eigenvalues determines the rank of Ψ, and the likelihood ratio test statistic for the rank is

$$LR_b = -T \sum_{i=b+1}^{n} log(1 - \lambda_i) \tag{17}$$

for the sequence of nested null hypotheses $H_{0,j-1} : b = j - 1$ $(j = 1, ..., n)$, starting with the most restricted model $b = 0$. These statistics are asymptotically distributed as $\chi^2_{(n-b)(m-b)}$, and we stop at the first hypothesis that cannot be rejected. Given the resulting estimate $\hat{b}$ we can estimate γ as the eigenvectors $V = (v_1 : ... : v_{\hat{b}})$ corresponding to the $\hat{b}$ largest eigenvalues λ_i $(i = 1, ..., \hat{b})$. As usual we can pick a just-identified estimate as $\hat{\gamma} = V(\gamma_0' V)^{-1}$, where $\gamma_0' = (I_{\hat{b}} : 0_{n-\hat{b}})$. Of course, the remaining parameters of the UVAR

Table 3: Co-breaking rank test, unrestricted breaks

H_0	eigenvalue	trace test statistic LR_b	p-value
$b = 0$	0.068	17.99	0.006
$b = 1$	0.006	1.47	0.479

Notes: The rank test procedure is described in section 4.1. The model used here has $n = 2$, $m = 3$.

(12) for a given estimate $\hat{\gamma}$ can then be estimated by a straightforward regression.

4.2 Empirical results

In our case, $n = 2$ and $m = 3$ initially, where however the status of the "extra" break in 1986 that we only found in the unemployment rate remains to be investigated. The concrete specification of model (10) is as follows: According to the previous estimates we set $T_1 = 1974q4$, $T_2 = 1986q4$, and $T_3 = 1996q2$, and we add the appropriate impulse dummies as described in the previous subsection. Furthermore, the slight discrepancies between the point estimates of the break dates in section 3 are also "dummied out" by filling the gaps with impulse dummies, thus effectively extending the non-modelled transition periods. Setting the lag length to $g = 3$ proved sufficient to account for the dynamics of the variables according to standard information criteria; the Hannan-Quinn and Schwarz criteria even indicated a lag length of only 2, but restricting the third lag was rejected by a LR test and also seemed overly strict for the given sample size and quarterly data. The effective sample is then 1948q4-2007q2, $T = 235$.

First we tested the full system with all three breaks for any co-breaking relationships. The test results in table 3 show that of course the existence of the breaks in the variables is significant (rank $b = 0$ is clearly rejected), but that there exists a co-breaking relation between the two variables unemployment and productivity growth (rank $b = 1$ cannot be rejected in favor of full rank 2, i.e. the co-breaking rank is accepted to be $n - b = 2 - 1 = 1$).

However, the normalized coefficients of the regime shift dummy "package" are esti-

Table 4: Co-breaking rank test

H_0	eigenvalue	trace test statistic LR_b	p-value
$b = 0$	0.067	17.5	0.008
$b = 1$	0.0047	1.11	0.573

Note: For this test no additional intercept for the post-1996 regime is allowed, i.e. the pre-1974 and post-1996 regimes are restricted to have equal intercepts. Therefore in terms of the co-breaking framework we have $m = 2$ here.

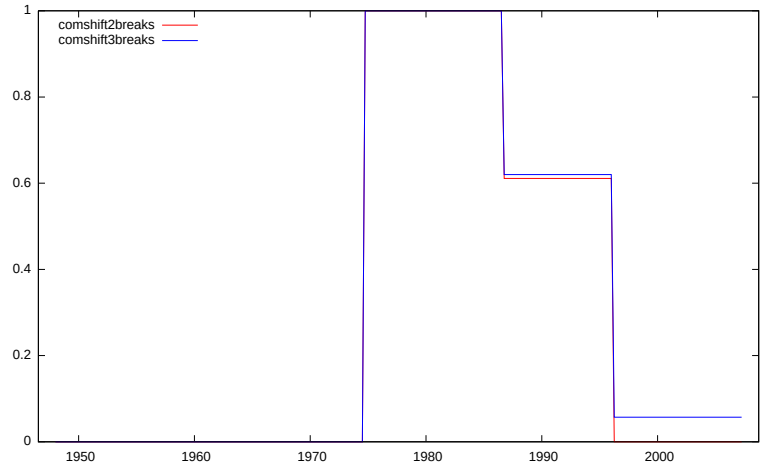

Figure 1: Comparison of unrestricted and restricted regime shift estimates. The blue line displays the unrestricted estimates with four effective regimes; for the red line the first and last regime are restricted to be equal.

mated as $[1, 0.62, 0.06]$, i.e., the additional intercept of the last (post-1996) regime is essentially zero, such that the current regime is virtually identical to the first (pre-1974) one. Therefore we also specify a system with only the first two shift dummies for 1974-1986 and 1986-1996, and the absence of the post-1996 shift dummy means that the pre-1974 and post-1996 regimes are restricted to have identical means. We re-run the co-breaking test, see table 4, and reach the same conclusion as before, namely that there exists a co-breaking relationship. Comparing the estimated regime shifts in the unrestricted and restricted systems in figure 1 shows that the difference is marginal. Diagnostic tests in table 5 show no problems of residual serial correlation. The presence of ARCH effects in the unemployment equation implies non-normality of the residuals, such that the system results are only quasi-ML estimates, but this does not invalidate the results.

Table 5: Diagnostic tests

equation	no auto-correlation (1-5)	normality	no ARCH (1-4)
unemployment	1.07 [0.376]	13.81 [0.001]	7.05 [0.000]
productivity growth	0.70 [0.625]	14.81 [0.001]	1.67 [0.159]
system test	0.85 [0.658]	25.77 [0.000]	-

Notes: These tests refer to the specification where the pre-1974 and post-1996 regimes are restricted to have identical means, thus the effective number of breaks is $m = 2$, and the co-breaking rank is $n - b = 1$. Numbers in square brackets are p-values. The tests for auto-correlation and ARCH effects are LM-type tests, calculated with PcGive.

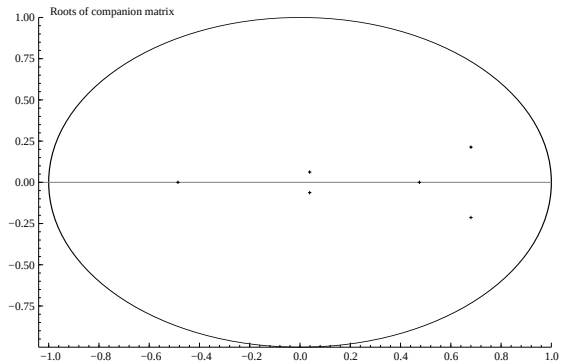

Figure 2: Roots of the system with common shifts. The system is restricted to have equal intercepts before 1974 and after 1996, as described before. The fact that the moduli of the roots are clearly smaller than unity implies that the series are $I(0)$ with breaks, not $I(1)$.

The obvious conclusion is that it is clearly acceptable to treat the first and final regimes as identical, such that they effectively form a joint common regime. Note that after having established that there are only two effective breaks in the system, the maximal number of variables without running into the "spurious co-breaking" problem is also two, given the arguments of section 4.1. Therefore, it would be impossible to add other covariates to the system that may be interesting on economic grounds.

Given the perennial controversy of how to model low-frequency movements of variables such as the unemployment rate it is also noteworthy that our system is stochastically stable, see the estimated roots of the companion matrix in figure 2. We therefore confirm the finding of Papell et al. (2000) that treating US unemployment as $I(1)$ (for example in Phillips curve models) does not seem to be completely adequate.

Now we use this restricted model to estimate the co-breaking vector as described above; the result is that we can write the steady-state relation between annual productivity growth (in percentages) and unemployment as:

$$\overline{u} = 9.84 - 1.76\overline{\Delta q} \tag{18}$$

In terms of the model in section 2 the estimate 1.76 corresponds to $\mu\lambda/\beta_u$, and if we assume $\mu\lambda = 1$ for the United States due to the findings discussed in Blanchard and Katz (1999) we obviously would have an estimate $\hat{\beta}_u = 0.57$. The system estimates of the long-run means are given as follows:

$$\begin{bmatrix} \overline{u} \\ \overline{\Delta q} \end{bmatrix} = \begin{bmatrix} \underset{(0.24)}{4.87} \\ \underset{(0.23)}{2.83} \end{bmatrix} + \begin{bmatrix} \underset{(0.48)}{2.48} \\ \underset{(0.45)}{-1.41} \end{bmatrix} \times \begin{cases} 1 & post74pre86 \\ 0.611 & post86pre96 \\ 0 & else \end{cases}$$

The two variables together with their estimated broken deterministic mean functions are shown in figures 3 and 4. Of course the reaction of unemployment to productivity growth in the short run can be radically different from what relation (18) suggests for the long run; for example, the most recent observations for both variables are below their estimated long-run means. Provided that no further shift occurs, both variables would therefore be expected to rise slightly over the medium term.

Finally, it is desirable to quantify the sampling uncertainty regarding the estimated regime means in some way. Unfortunately it is not trivial to provide confidence intervals for the common shifts under the imposed co-breaking restriction. As a quick-and-dirty substitute we therefore provide the following estimates, which we call semi-restricted: Namely, we impose that the pre-1974 and post-1996 regimes have equal means, but we do not impose the reduced rank of the co-breaking matrix. This specification can be estimated simply as (a set of) VARs augmented by regime shift dummies, and confidence intervals of the long-run impact of the regime means are available in PcGive. The obvious drawback is that the point estimates of the regime-dependent means may not exactly

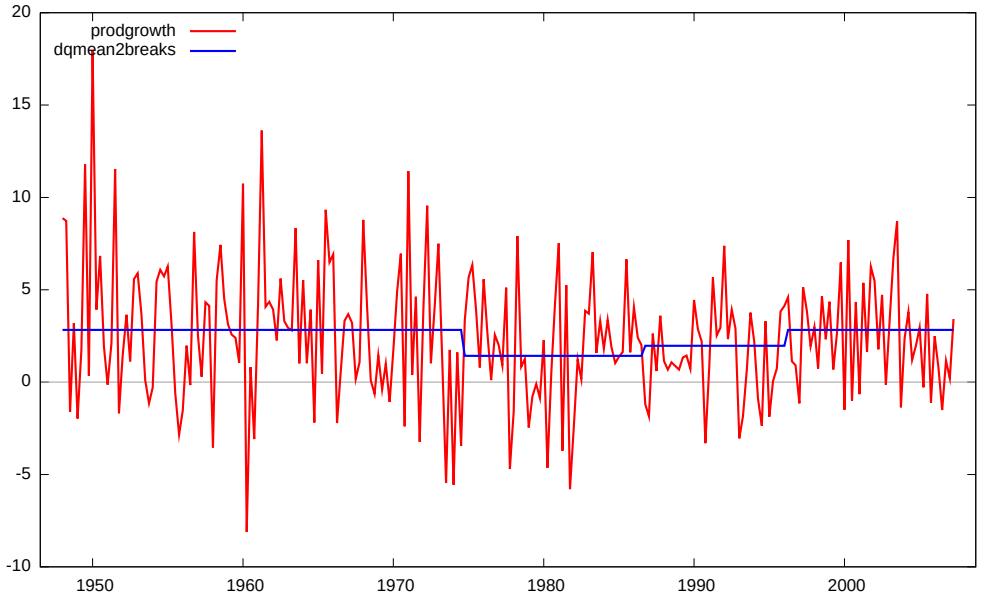

Figure 3: Estimates of productivity growth regimes

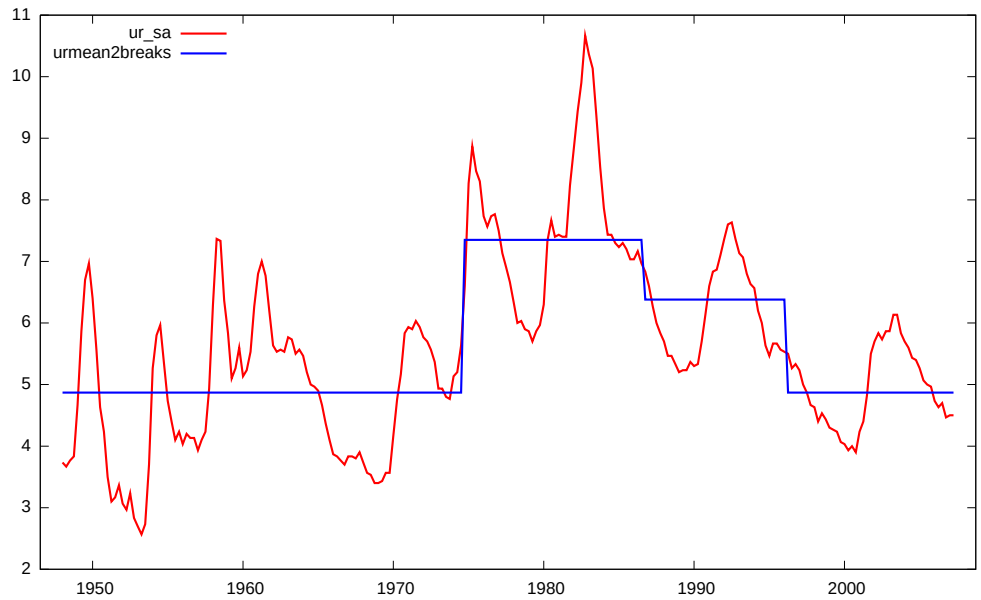

Figure 4: Estimates of unemployment rate regimes

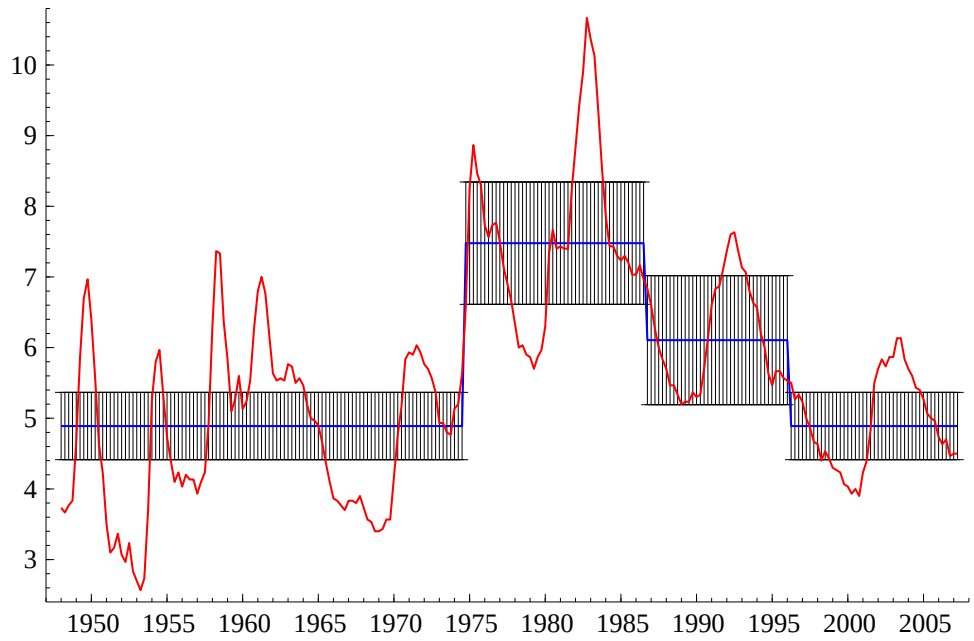

Figure 5: Semi-restricted unemployment regime estimates with error bands. The pre-1974 and post-1996 regimes are restricted to have equal means, but in this specification the previously estimated co-breaking relationship is not imposed, instead the 1974-1986 and 1986-1996 regime shift dummies are included unrestrictedly. This facilitates calculating the confidence intervals for the estimated regime-dependent means.

match the previously estimated co-breaking relation. The resulting semi-restricted estimates with the corresponding confidence bands are shown in figures 5 and 6. Note that overlapping confidence intervals across regimes do not imply that the regime means are equal (up to statistical error), because for such a test the covariance across regimes would have to be taken into account as well.

The most striking feature is the high uncertainty around the mean of productivity growth in the regime between 1986 and 1996, which mirrors the previous univariate findings that only two breaks were found to be significant in productivity growth alone, namely 1974 and 1996. Here it is really only the unemployment rate which provided the necessary information to estimate the amount of the shift in 1986 in the system.

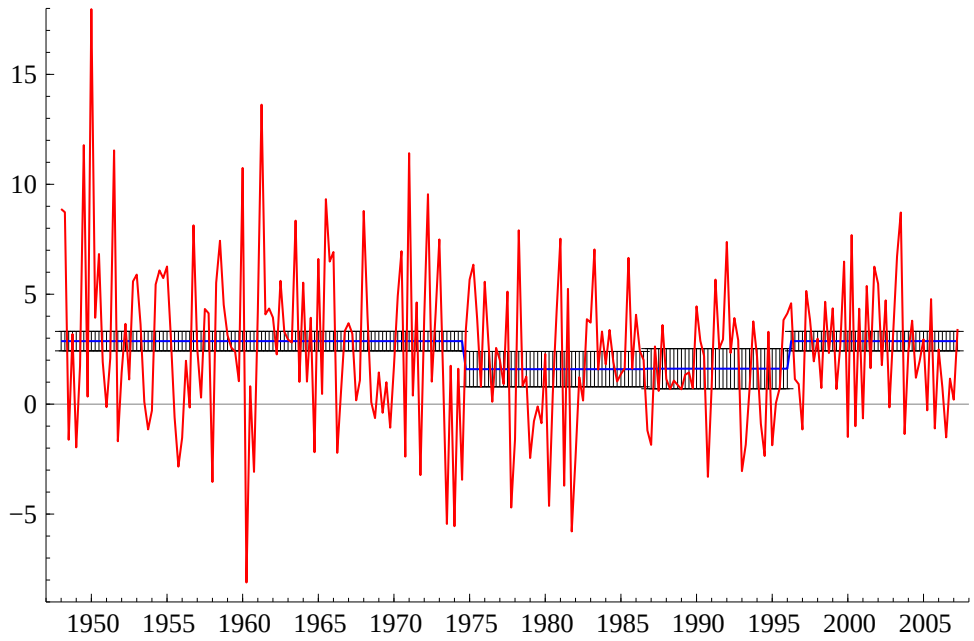

Figure 6: Semi-restricted productivity growth regime estimates with error bands. See figure 5 for explanations.

5 Conclusions

This paper has dealt with the long-run connection between labor productivity growth and unemployment, exploiting the well-known slowdown of productivity growth in the 1970s and the later speed-up in the 1990s, which were treated as exogenous determinants for unemployment developments. Our results indicate that a "co-breaking" framework is a natural and empirically adequate model to capture the long-run link between productivity growth and unemployment in the United States. Such a framework models the non-stationarity in the individual variables through infrequent shifts in their means, i.e. in the so-called deterministic component. These shifts are common to both variables; however, at medium to high frequencies the properties of the variables are quite different, namely high persistence in unemployment and little serial correlation in productivity growth. As a consequence of the *common* shifts there exists a long-run relation as a linear combination of the variables that is free from mean-shifts; our estimates imply a negative long-run connection between productivity growth and unemployment. Unfortunately a necessary

condition for a co-breaking analysis as in the present paper is that the number of variables must not exceed the number of breaks ($n \leq m$), which is a severe restriction. But of course it is possible to add stationary variables to a system once co-breaking has been established.

An additional finding was that the productivity speedup after 1996 restored pre-1974 growth rates (about 2.8 percent) and therefore also pulled mean unemployment back to its original level (roughly 5 percent). This result suggests that the regimes from 1974 to 1996 constituted a historical exception, while the pre-1974 and post-1996 regimes represent the normal workings of the US economy. Greenwood and Yorukoglu (1997) for example have argued that during the slow but steady diffusion of computers and associated business practices through all sectors of an economy the measured productivity growth will be lower as usual until the adoption is complete.[13] Our empirical findings are consistent with this explanation of the productivity slowdown.

Finally it may be worthwhile to point out that our results of significant shifts in these variables also carry implications for other empirical applications. For example, Phillips curves estimates must account for the decline of equilibrium unemployment in the 1990s to avoid mis-specification.

References

Acemoglu, D. (2003). Labor- and capital-augmenting technical change. *Journal of the European Economic Association*, 1(1):1–37.

Aghion, P. and Howitt, P. (1994). Growth and unemployment. *Review of Economic Studies*, 61:477–494.

Bai, J. and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, 66(1):47–78.

Bai, J. and Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, 18:1–22.

[13] See, e.g., Jorgenson (2001) and references therein for a detailed account of the underlying evidence.

Ball, L. and Mankiw, N. G. (2002). The NAIRU in theory and practice. *Journal of Economic Perspectives*, 16(4):115–136.

Blanchard, O. and Katz, L. F. (1999). Wage dynamics: Reconciling theory and evidence. *American Economic Review Papers&Proceedings*, 89(2):69–74.

Bradbury, K. (2005). Additional slack in the economy: The poor recovery in labor force participation during this business cycle. Public Policy Briefs 05-2, Federal Reserve Bank of Boston.

Doornik, J. A. and Hendry, D. F. (2001). *Modelling Dynamic Systems Using PcGive 10*. Timberlake Consultants Ltd.

Durlauf, S. N. and Quah, D. T. (1999). *The New Empirics of Economic Growth*, volume 1a of *Handbook of Macroeconomics*, pages 235–297. J.B. Taylor and M. Woodford.

Engle, R. and Kozicki, S. (1993). Testing for common features. *Journal of Business Economics and Statistics*, 11:369–395.

Evans, G. W. (1989). Output and unemployment dynamics in the united states: 1950-1985. *Journal of Applied Econometrics*, 4(3):213–237.

Fernald, J. (2007). Trend breaks, long-run restrictions, and contractionary technology improvements. *Journal of Monetary Economics*, 54(8):2467–2485.

Greenwood, J. and Yorukoglu, M. (1997). 1974. *Carnegie-Rochester Conference Series on Public Policy*, 46:49–95.

Hansen, B. E. (2001). The new econometrics of structural change: Dating breaks in U.S. labor productivity. *Journal of Economic Perspectives*, 15(4):117–128.

Hendry, D. F. and Massmann, M. (2007). Co-breaking: Recent advances and a synopsis of the literature. *Journal of Business and Economic Statistics*, 25:33–51.

Hogan, V. (2004). Wage aspirations and unemployment persistence. *Journal of Monetary Economics*, 51:1623–1643.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press.

Johansen, S., Mosconi, R., and Nielsen, B. (2000). Cointegration analysis in the presence of structural breaks in the deterministic trend. *The Econometrics Journal*, 3(2):216–249.

Jorgenson, D. W. (2001). Information technology and the u.s. economy. *American Economic Review*, 91:1–32.

Krolzig, H.-M. and Toro, J. (2002). Testing for super-exogeneity in the presence of common deterministic shifts. *Les Annales d'Economie et de Statistique*, (67-68).

Mortensen, D. and Pissarides, C. A. (1998). Technological progress, job creation and job destruction. *Review of Economic Dynamics*, 1:733–753.

Muscatelli, V. A. and Tirelli, P. (2001). Unemployment and growth: Some empirical evidence from structural time series models. *Applied Economics*, 33:1083–1088.

OECD (1997). *Recent Labour Market Developments and Prospects*, chapter 1, pages 1–25. OECD, Paris.

Papell, D. H., Murray, C. J., and Ghiblawi, H. (2000). The structure of unemployment. *Review of Economics and Statistics*, 82(2):309–315.

Pissarides, C. A. and Vallanti, G. (2007). The impact of TFP growth on steady-state unemployment. *International Economic Review*, 48(2):607–640.

Prat, J. (2007). The impact of disembodied technological progress on unemployment. *Review of Economic Dynamics*, 10:106–125.

Qu, Z. and Perron, P. (2007). Estimating and testing structural changes in multivariate regressions. *Econometrica*, 75(2):459–502.

Saikkonen, P. and Lütkepohl, H. (2000). Testing for the cointegrating rank of a VAR process with structural shifts. *Journal of Business and Economic Statistics*, 18(4):451–464.

Stiglitz, J. (1997). Reflections on the natural rate hypothesis. *Journal of Economic Perspectives*, 11:3–10.

Tripier, F. (2006). Sticky prices, fair wages, and the co-movements of unemployment and labor productivity growth. *Journal of Economic Dynamics & Control*, 30:2749–2774.

U.S.-Government (2004). Unemployment compensation, federal-state partnership. Technical report, Department of Labor.

Valletta, R. (2005). Help-wanted advertising and job vacancies. Economic Letter 2005-02, Federal Reserve Bank San Francisco.