The quest for Earth's oldest ice *p. 436*

Origin of lung disease in cystic fibrosis *p. 503*

Dispensing with genes on the Y chromosome *p. 514*

Science

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Jupiter rising

Geometric techniques in Babylonian astronomy

pp. 435 & 482

ARCHAEOLOGY

Ancient Babylonians took first steps to calculus

Math whizzes left precocious geometric calculations on clay tablets by 50 B.C.E.

By **Ron Cowen**

Tracking and recording the motion of the sun, the moon, and the planets as they paraded across the desert sky, ancient Babylonian astronomers used simple arithmetic to predict the positions of celestial bodies. Now, new evidence reported on p. 482 reveals that these astronomers, working several centuries B.C.E., also employed sophisticated geometric methods that foreshadow the development of calculus. Historians had thought such techniques did not emerge until more than 1400 years later, in 14th century Europe.

The study "is an extremely important contribution to the history of Babylonian astronomy, and more generally to the history of science," says astronomy historian John Steele of Brown University, who was not part of the work.

Astroarchaeologist Mathieu Ossendrijver of Humboldt University in Berlin bases his findings on a reexamination of clay tablets, one of them unknown until recently, dating from 350 B.C.E. to 50 B.C.E. One week each year for the past 14 years, Ossendrijver has made a pilgrimage to the British Museum's vast collection of tablets inscribed in the Babylonian cuneiform script. He was trying to solve a puzzle posed by two tablets dealing with astronomical calculations: They also contained instructions for constructing a trapezoidal figure that seemed unrelated to anything astronomical.

Between 2002 and 2008, Ossendrijver, an astrophysicist turned historian, studied two other tablets that also prescribed the drawing of a trapezoid, and in these he thought he could make out a reference to Jupiter. The giant planet was a favorite among the Babylonians, who equated the orb with their main god, Marduk, patron deity of the city of Babylon. But the Jupiter link was tentative.

Then, late in 2014, retired Assyriologist Hermann Hunger of the University of Vienna visited Ossendrijver, bringing photos taken decades ago of an uncatalogued Babylonian tablet from the British Museum that described some kind of astronomical computation. Alone in his office a few months later, Ossendrijver perused the photos. The images were blurry and the inscriptions slanted, making them hard to read, but he realized the numbers were identical to those in the trapezoid inscriptions he had been scrutinizing. By comparing the photos with fragments of other Babylonian texts, he discovered that the computations described the motion of Jupiter.

Examining all of the tablets at the British Museum, Ossendrijver figured out that the trapezoid calculations were a tool for calculating Jupiter's displacement each day along the ecliptic, the path that the sun appears to trace through the stars. The computations recorded on the tablets covered a period of 60 days, beginning on a day when the giant planet first appeared in the night sky just before dawn.

Marduk, the patron god of Babylon, was equated with Jupiter, so ancient astronomers charted the planet's path across the heavens with care.

PHOTO: © AS400 DB/CORBIS

During that interval, Jupiter's motion across the sky appears to slow. (Such erratic apparent motion stems from the complex combination of Earth's own orbit around the sun with that of Jupiter.) A graph of Jupiter's apparent velocity against time slopes downward, so that the area under the curve forms a trapezoid. The area of the trapezoid in turn gives the distance that Jupiter has moved along the ecliptic during the 60 days. Calculating the area under a curve to determine a numerical value is a basic operation, known as the integral between two points, in calculus. Discovering that the Babylonians understood this "was the real 'aha!' moment," Ossendrijver says.

Although elated, Ossendrijver wasn't ready to publish, because a second part of the trapezoid prescription remained unclear. By delving into older, purely mathematical Babylonian texts written between 1800 B.C.E. and 1600 B.C.E., which also described computations with a trapezoid, he realized that the astronomers who made the tablets had gone a step further. To compute the time at which Jupiter would have moved halfway along its ecliptic path, the astronomers divided the 60-day trapezoid into two smaller ones of equal area. The vertical line dividing the two trapezoids marked the halfway time; because of the different shapes of the trapezoids, it indicated not 30 days but slightly fewer.

The Babylonians had developed "abstract mathematical, geometrical ideas about the connection between motion, position and time that are so common to any modern physicist or mathematician," Ossendrijver says.

Indeed, compared with the complex geometry embraced by the ancient Greeks a few centuries later, with its cycles and epicycles, the inscriptions reflect "a more abstract and profound conception of a geometrical object in which one dimension represents time," says historian Alexander Jones of New York University in New York City. "Such concepts have not been found earlier than in 14th century European texts on moving bodies," he adds. "Their presence … testifies to the revolutionary brilliance of the unknown Mesopotamian scholars who constructed Babylonian mathematical astronomy."

After cuneiform died out around 100 C.E., Babylonian astronomy was thought to have been virtually forgotten, he notes. It was left to French and English philosophers and mathematicians in the late Middle Ages to reinvent what the Babylonians had developed.

The new discovery may hint that Babylonian geometry did not die out completely after all. Either way, Jones says, learning how the Babylonians astronomers acquired their geometric acumen "would tell us something about why human beings do science in the first place, and from time to time do it very well indeed." ■

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eastern tropical Pacific and Antarctica peaked during each of the last two glacial terminations (*28*), consistent with the timing of enhanced EPR hydrothermal activity.

Isolating a mechanistic linkage between ridge magmatism and glacial terminations will require a suite of detailed proxy records from multiple ridges that are sensitive to mantle carbon and geothermal inputs, as well as modeling studies of their influence in the ocean interior. The EPR results establish the timing of hydrothermal anomalies, an essential prerequisite for determining whether ridge magmatism can act as a negative feedback on ice-sheet size. The data presented here demonstrate that EPR hydrothermal output increased after the two largest glacial maxima of the past 200,000 years, implicating mid-ocean ridge magmatism in glacial terminations.

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SUPPLEMENTARY MATERIALS

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Materials and Methods
Supplementary Text
Figs. S1 to S11
Tables S1 to S5
References (*31–45*)

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HISTORY OF SCIENCE

Ancient Babylonian astronomers calculated Jupiter's position from the area under a time-velocity graph

Mathieu Ossendrijver*

The idea of computing a body's displacement as an area in time-velocity space is usually traced back to 14th-century Europe. I show that in four ancient Babylonian cuneiform tablets, Jupiter's displacement along the ecliptic is computed as the area of a trapezoidal figure obtained by drawing its daily displacement against time. This interpretation is prompted by a newly discovered tablet on which the same computation is presented in an equivalent arithmetical formulation. The tablets date from 350 to 50 BCE. The trapezoid procedures offer the first evidence for the use of geometrical methods in Babylonian mathematical astronomy, which was thus far viewed as operating exclusively with arithmetical concepts.

The so-called trapezoid procedures examined in this paper have long puzzled historians of Babylonian astronomy. They belong to the corpus of Babylonian mathematical astronomy, which comprises about 450 tablets from Babylon and Uruk dating between 400 and 50 BCE. Approximately 340 of these tablets are tables with computed planetary or lunar data arranged in rows and columns (*1*). The remaining 110 tablets are procedure texts with computational instructions (*2*), mostly aimed at computing or verifying the tables. In all of these texts the zodiac, invented in Babylonia near the end of the fifth century BCE (*3*), is used as a coordinate system for computing celestial positions. The underlying algorithms are structured as branching chains of arithmetical operations (additions, subtractions, and multiplications) that can be represented as flow charts (*2*). Geometrical concepts are conspicuously absent from these texts, whereas they are very common in the Babylonian mathematical corpus (*4*–*7*). Currently four tablets, most likely written in Babylon between 350 and 50 BCE, are known to preserve portions of a trapezoid procedure (*8*). Of the four procedures, here labeled B to E (figs. S1 to S4), one (B) preserves a mention of Jupiter and three (B, C, E) are embedded in compendia of procedures dealing exclusively with Jupiter. The previously unpublished text D probably belongs to a similar compendium for Jupiter. In spite of these indications of a connection with Jupiter, their astronomical significance was previously not acknowledged or understood (*1*, *2*, *6*).

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A recently discovered tablet containing an unpublished procedure text, here labeled text A (Fig. 1), sheds new light on the trapezoid procedures. Text A most likely originates from the same period and location (Babylon) as texts B to E (*8*). It contains a nearly complete set of instructions for Jupiter's motion along the ecliptic in accordance with the so-called scheme X.S$_1$ (*2*). Before the discovery of text A, this scheme was too fragmentarily known for identifying its connection with the trapezoid procedures. Covering one complete synodic cycle, scheme X.S$_1$ begins with Jupiter's heliacal rising (first visible rising at dawn), continuing with its first station (beginning of apparent retrograde motion), acronychal rising (last visible rising at dusk), second station (end of retrograde motion), and heliacal setting (last visible setting at dusk) (*2*). Scheme X.S$_1$ and the four trapezoid procedures are here shown to contain or imply mathematically equivalent descriptions of Jupiter's motion during the first 60 days after its first appearance. Whereas scheme X.S$_1$ employs a purely arithmetical terminology, the trapezoid procedures operate with geometrical entities.

In text A, Jupiter's motion along the ecliptic is described in terms of its daily displacement (modern symbol: v) expressed in °/d (degrees/day) and its total displacement (S) expressed in degrees. A crucial new insight about scheme $X.S_1$ provided by text A concerns its use of piecewise linearly changing values for v. Although not formulated explicitly, this linear dependence on time is clearly implied (*8*). Jupiter's motion along the ecliptic is described for two consecutive intervals of 60 days between its first appearance and its first station. For each interval, initial and final values of v are provided. Note that Babylonian astronomy employs a sexagesimal; i.e., base-60 place-value system in which numbers are represented as sequences of digits between 0 and 59, each associated with a power of 60 that decreases in the right direction. In the commonly used modern notation for these numbers, all digits are separated by commas, except for the digit pertaining to 60°, which is separated from the next one pertaining to 60^{-1} by a semicolon (;), the analog of our decimal point. For the first interval of 60 days, $v_0 = 0;12°/d$ (=12/60) and $v_{60} = 0;9,30°/d$ ($=9/60 + 30/60^2$). Their sum is multiplied by 0;30 (=1/2), resulting in a mean value $(v_0 + v_{60})/2 = 0;10,45°/d$, which is multiplied by 1,0 (=60) days, resulting in a total displacement $S = 1,0\bullet(v_0 + v_{60})/2 = 10;45°$. For the second interval, $v_{60} = 0;9,30°/d$ and $v_{120} = 0;1,30°/d$ ($=1/60 + 30/60^2$), leading to $(v_{60} + v_{120})/2 = 0;5,30°/d$ and $S = 5;30°$. The sum of the total displacements, $10;45° + 5;30° = 16;15°$, is declared to be the total distance by which Jupiter proceeds along the ecliptic in 120 days. In other words, the ecliptic longitude of Jupiter after 60 and 120 days is computed as $\lambda_{60} = \lambda_0 + 10;45°$ and $\lambda_{120} = \lambda_0 + 16;15°$, respectively.

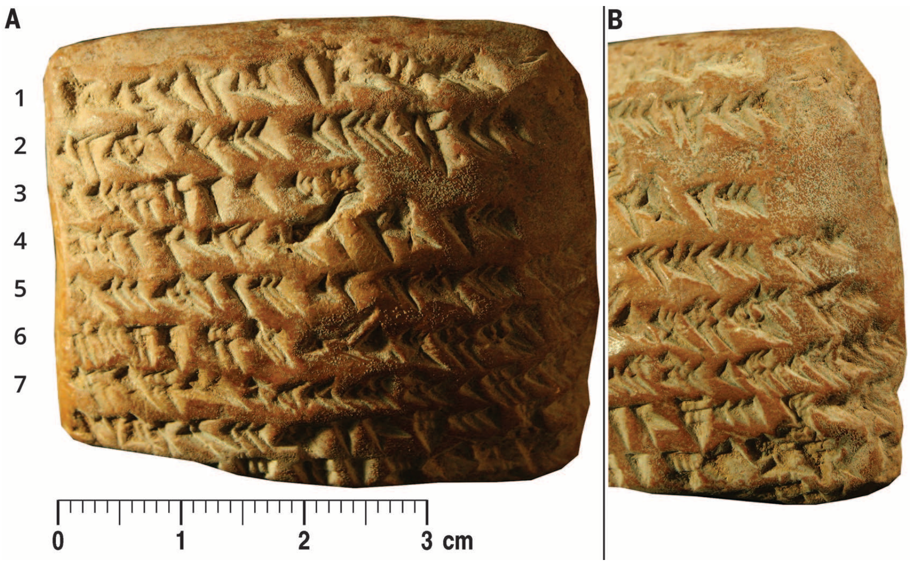

Fig. 1. Photograph of text A (lines 1 to 7). (**A**) Full image. (**B**) Partial image of the right side taken under different lighting conditions.

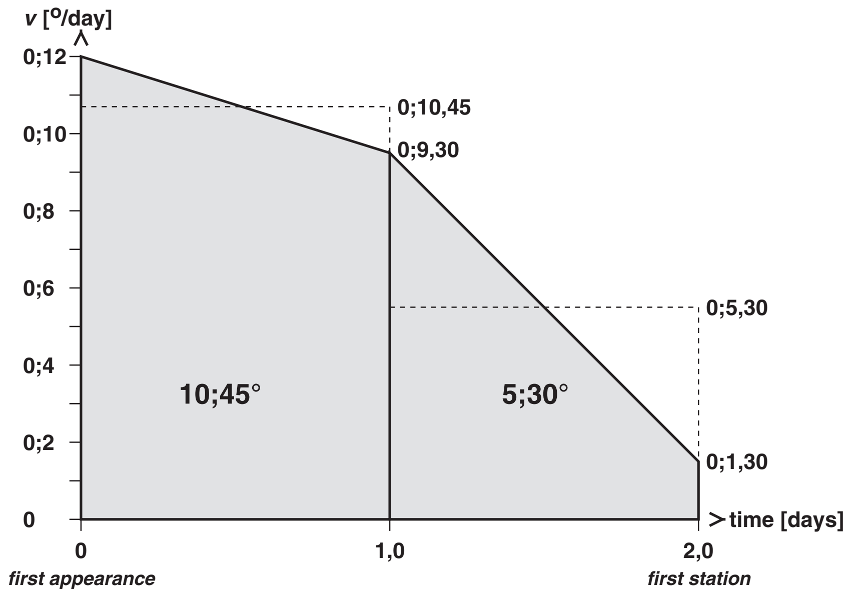

Fig. 2. Time-velocity graph of Jupiter's motion. Daily displacement along the ecliptic (v) between Jupiter's first appearance (day 0) and its first station (day 120) as a function of time according to scheme $X.S_1$ as inferred from text A. All numbers and axis labels are in sexagesimal place-value notation. The areas of the trapezoids, 10;45° and 5;30°, each represent Jupiter's total displacement during one interval of 60 days.

Text A does not describe how v varies from day to day, but of the three forms of time dependence of v that are attested in Babylonian planetary texts—piecewise constant, linear, or quadratic in each time interval (*2*, *9*)—only the linear one comes into question. If v were piecewise constant, then S should equal $60\bullet v$ for each interval. If v were piecewise quadratic, then $S = 60\bullet(v_0 + v_{60})/2$ can only be some rough approximation. That would be unexpected, since other tablets imply that some Babylonian scholars in this period were familiar with the exact algorithm for summing a quadratic series (*9*, *10*). By contrast, the values of S computed in text A are exact if one assumes that v changes linearly in each interval. It follows that in scheme $X.S_1$, v decreases linearly from 0;12°/d to 0;9,30°/d between day 0 and day 60, and from 0;9,30°/d to 0;1,30°/d between day 60 and day 120.

This new reconstruction of the first 120 days of scheme $X.S_1$ results in trapezoidal figures if v is plotted against time in a modern fashion (Fig. 2). It is important to note that text A itself does not contain or imply a geometrical representation. However, it turns out to be explicitly formulated in the trapezoid procedures, texts B to E (figs. S1 to S4). Although their formulation differs in details, at least three of them (B to D) consist of the same two parts, I and II.

In part I, Jupiter's total displacement for the first 60 days of scheme $X.S_1$ is computed. A corresponding introductory statement mentioning Jupiter and the measures of the trapezoid is partly preserved in texts B and C, and perhaps in text E (*8*). The number 10;45, referred to as the "area" of the trapezoid (B, C), is then added to the "position of appearance" (B, C, D), the technical term for Jupiter's ecliptical longitude at first appearance, i.e., $\lambda_{60} = \lambda_0 + 10;45°$. Texts B and C partly preserve the computation of 10;45 as the area of the trapezoid through a series of steps equivalent to the computations in text A. Its "large side" and "small side," $v_0 = 0;12°/d$ and $v_{60} = 0;9,30°/d$, are averaged, $(v_0 + v_{60})/2 = 0;10,45°/d$, which is then multiplied by 60 days, the width of the trapezoid, resulting in 10;45°. The latter operation is partly preserved in text C and can be restored in text B.

Part II, partly preserved in texts B, D, and E, is concerned with the time in which Jupiter reaches a position referred to by a term tentatively translated as the "crossing" (*8*). It is now clear that this denotes a point on the ecliptic, say λ_c, located halfway between λ_0 and λ_{60}, i.e., $\lambda_c = \lambda_0 + 10;45°/2$. This interpretation is consistent with a statement, preserved only in text B, according to which the "crossing" is located in the middle of Jupiter's "path," readily interpreted as a reference to the ecliptical segment from λ_0 to λ_{60}. Texts B and D also preserve the following statement that

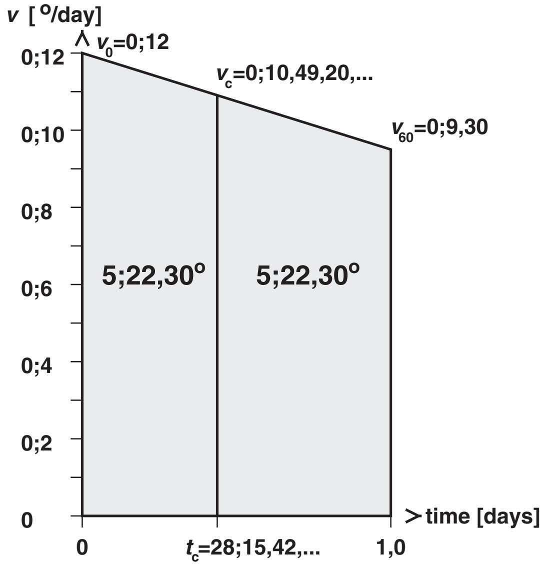

Fig. 3. Partitioning the trapezoid for days 0 to 60. The time at which Jupiter reaches the "crossing," t_c, where it has covered the distance 5;22,30° = 10;45°/2, is computed geometrically by dividing the trapezoid for days 0 to 60 into two smaller trapezoids of equal area. In text E, v_c is rounded to 0;10,50°/d, resulting in t_c = 28 d, S_1 = 5;19,40°, t_2 = 32 d, and S_2 = 5;25,20°.

precedes the solution procedure: "Concerning this 10;45, you see when it is halved." The time in which Jupiter reaches λ_c, say t_c, is then computed by the following geometrical method: The trapezoid for days 0 to 60 is divided into two smaller trapezoids of equal area (Fig. 3). In order to achieve this, the Babylonian astronomers applied a partition procedure that is well-attested in Old Babylonian (2000 to 1800 BCE) mathematics (*5*, *6*). In modern terms, it can be formulated as follows: If v_0 and v_{60} are the parallel sides of a trapezoid, then the intermediate parallel that divides it into two trapezoids of equal area has a height $v_c = [(v_0{}^2 + v_{60}{}^2)/2]^{1/2}$. In the present case, v_c denotes Jupiter's daily displacement when it is at the "crossing." This expression follows from equating the areas of the partial trapezoids, $S_1 = t_c \bullet (v_0 + v_c)/2 = S_2 = t_2 \bullet (v_c + v_{60})/2$, where t_c and t_2 are the widths of these trapezoids, and using $t_c = t \bullet (v_0 - v_c)/(v_0 - v_{60})$, where $t = t_c + t_2$ is the width of the original trapezoid (*6*, *10*). Inserting v_0 = 0;12°/d, v_{60} = 0;9,30°/d, and t = 1,0 d, we obtain $v_c = [(0;2,24 + 0;1,30,15)/2]^{1/2} = (0;1,57,7,30)^{1/2}$ = 0;10,49,20,44,58,...°/d, t_c = 28;15,42,0,48,...d, and t_2 = 31;44,17,59,12,...d. The computation of v_c is partly preserved in text D up to the addition 0;2,24 + 0;1,30,15 (*8*). In text B, the related quantity $u^2 = (v_0{}^2 - v_{60}{}^2)/2$ = (0;2,24 − 0;1,30,15)/2 = 0;0,26,52,30 is computed. This was most likely followed by another step in which v_c was computed using $v_c{}^2 = v_0{}^2 - u^2$. Whereas all known Old Babylonian examples of the partition algorithm concern trapezoids for which v_c, v_0, and v_{60} are terminating sexagesimal numbers (*6*), the present solution does not terminate in the sexagesimal system. Hence, texts B to E can only have offered rounded results for v_c and t_c. Nothing remains of this in texts B to D, but text E partly preserves a computation involving 0;10,50, which is, most plausibly, an approximation of v_c. This interpretation is confirmed by the fact that text E also mentions the value t_c = 28 d and, very likely, t_2 = 32 d, both in exact agreement with $t_c = 60 \bullet (v_0 - v_c)/(v_0 - v_{60})$ and $t_2 = 60 - t_c$ if one approximates v_c = 0;10,50°/d. By rounding v_c, only an approximately equal partition of the trapezoid is achieved.

Also partly preserved in text E is a computation of the area of the second partial trapezoid, using the same method as before, leading to $S_2 = t_2 \bullet (v_c + v_{60})/2$, where t_2 = 32 days, v_c = 0;10,50°/d, and v_{60} = 0;9,30°/d. The value of S_2 is broken away but can be restored as 5;25,20°. The probable purpose of this computation was to verify the solution for v_c, as is done in the Old Babylonian mathematical text UET 5, 858 (*5*, *11*). The analogous computation of the area of the first partial trapezoid, which can be reconstructed as $S_1 = t_c \bullet (v_0 + v_c)/2$ = 5;19,40°, is not preserved. Neither of these values equals 5;22,30° = $S/2$ as they ideally should (Fig. 3), a direct consequence of the rounding of v_c to 0;10,50°/d. At most two more lines are partly preserved in texts B, D, and E, but they are too fragmentary for an interpretation.

The evidence presented here demonstrates that Babylonian astronomers construed Jupiter's displacement along the ecliptic during the first 60 days after its first appearance as the area of a trapezoid in time-velocity space. Moreover, they computed the time when Jupiter covers half this distance by partitioning the trapezoid into two smaller ones of ideally equal area. These computations predate the use of similar techniques by medieval European scholars by at least 14 centuries. The "Oxford calculators" of the 14th century CE, who were centered at Merton College, Oxford, are credited with formulating the "Mertonian mean speed theorem" for the distance traveled by a uniformly accelerating body, corresponding to the modern formula $s = t \bullet (v_0 + v_1)/2$, where v_0 and v_1 are the initial and final velocities (*12*, *13*). In the same century Nicole Oresme, in Paris, devised graphical methods that enabled him to prove this relation by computing s as the area of a trapezoid of width t and heights v_0 and v_1 (*12*). Part I of the Babylonian trapezoid procedures can be viewed as a concrete example of the same computation. They also show that Babylonian astronomers did, at least occasionally, use geometrical methods for computing planetary positions. Ancient Greek astronomers such as Aristarchus of Samos, Hipparchus, and Claudius Ptolemy also used geometrical methods (*12*), while arithmetical methods are attested in the Antikythera mechanism (*14*) and in Greco-Roman astronomical papyri from Egypt (*15*). However, the Babylonian trapezoid procedures are geometrical in a different sense than the methods of the mentioned Greek astronomers, since the geometrical figures describe configurations not in physical space but in an abstract mathematical space defined by time and velocity (daily displacement).

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SUPPLEMENTARY MATERIALS

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Materials and Methods
Figs. S1 to S4
References (*16*–*21*)

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Supplementary Materials for

Ancient Babylonian astronomers calculated Jupiter's position from the area under a time-velocity graph

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This PDF file includes:

Materials and Methods
Figs. S1 to S4
References

Materials and Methods

Transliterations, translations and photographs of the cuneiform tablets

Cuneiform Texts A-E were transliterated and translated from the original clay tablets during several visits to the Study Room of the Middle Eastern department of the British Museum (London) between 2002 and 2015 and from photographs made on these occasions. All tablets and fragments are identified by registration numbers assigned by the British Museum (BM). The transliterations were made in accordance with Assyriological conventions, i.e. logograms are written in capitals, Akkadian phonetic signs in italics, [x] indicates a sign broken away, [...] a break of unknown length, and ⸢x⸣ a damaged sign. In the translations, missing text is indicated by [...], untranslatable text by .... Note that the cuneiform notation for sexagesimal numbers lacks an equivalent of our decimal point. In the transliterations this feature is maintained by separating all digits by a period (.). In the translations the absolute value of each number, as inferred from the context, is indicated by separating the digit pertaining to 60^0 from the next one pertaining to 60^{-1} by a semicolon (;) and all other digits by commas. Also note that initial and final vanishing digits are not written in cuneiform. For instance, in line 1 of Text A the number written as 12 represents 0;12, which stands for 12/60, 1 represents 1,0 = 60, and 9.30 represents 0;9,30 = $9/60+30/60^2$. All displacements of Jupiter in Texts A-E are measured in degrees (º), but this unit is, as usual in Babylonian mathematical astronomy, not mentioned explicitly.

Provenance and date of Texts A-E

The tablets on which Texts A-E are written were excavated unscientifically in Iraq in the 19th century, along with thousands of other tablets (*16*). Even though their exact findspot is not documented, there is a consensus that the astronomical tablets from these excavations originate from Babylon, the main center of Babylonian astronomy during the first Millennium BCE (*2, 16*). This is confirmed by occasional references to Babylon's main temple on some of the astronomical tablets (*1, 2*).

Due to a combination of factors it is impossible to assign very precise dates to Texts A-E. First, they lack a colophon that might have mentioned a date of writing or the name of a datable astronomer. Second, they do not mention datable astronomical phenomena. Third, since they were excavated unscientifically no information about the archaeological stratigraphy of the tablets is available. However, the range of possible dates is constrained by the following considerations. First, the computations in Texts A-E employ ecliptical coordinates, which implies that they were written after the estimated date of introduction of these coordinates near the end of the fifth century BCE (*3*). Second, many of the other tablets from Babylon that deal with mathematical astronomy have been dated. The dates of these tablets roughly extend from 350 BCE to 50 BCE, while their distribution peaks between 180 BCE and 100 BCE (*1*). Texts A-E therefore very likely date from 350-50 BCE, the range of most probable dates being 180-100 BCE.

Text A

Text A is inscribed, in a highly cursive hand, on the tablet BM 40054, which measures 4.3 x 3.8 x 2.0 cm (Fig. 1). The tablet arrived in the British Museum in 1881, having been excavated unscientifically in Iraq, very likely in Babylon (*16*). The tablet is nearly intact, apart from a slice of clay, perhaps corresponding to one line of text, that is missing from the lower edge of the obverse and the upper edge of the reverse. The surface of the tablet is notably curved. The tablet contains a single procedure concerning scheme X.S_1 for Jupiter. It was previously unpublished. Text A comprises lines 1-7 of the obverse; the remaining approximately 12 lines deal with subsequent intervals of Jupiter's synodic cycle and will be published elsewhere.

Transliteration

1 ME IGI 12 EN 1 ME 9.30

2 12 *u*$_3$ 9.30 21.30 A 30

3 10.45 A 1 10.4⸢5⸣

4 TA 1 ME GI ⸢EN⸣ 1 ME 1.30

5 9.30 *u*$_3$ 1.30 11 A.RA$_2$ 30 5.30

6 5.30 A 1 5.30 (*erasure*) 10.45 *u*$_3$ 5.30

7 ⸢16⸣.15 PAP.PAP TA IGI EN UŠ 16.15 DU.DU?

Translation

1 The day when it appears: 0;12, until 1,0 days, 0;9,30.

2 0;12 and 0;9,30 is 0;21,30, times 0;30

3 is 0;10,45, times 1,0 is 10;45.

4 After completing 1,0 days, until 1,0 days 0;1,30.

5 0;9,30 and 0;1,30 is 0;11, times 0;30 is 0;5,30.

6 0;5,30 times 1,0 days is 5;30. (*erasure*) 10;45 and 5;30 is

7 16;15, the total. From appearance until station the motion is 16;15.

Philological remarks

2, 3, 6) In these lines the usual logogram A.RA$_2$, „times“, is abbreviated to A.

6) After 5.30 there is an erased sign that was overwritten by the 10.

7) DU.DU?: the sign following DU is written on the edge in a crammed manner. It looks like BI with a horizontal wedge below it. This sign is here assumed to be a deformed DU.

Like DU alone, the logogram DU.DU? probably represents *alāku*, „course; motion", or a related noun derived from this verb. Alternatively it may represent the present tense of this verb, *illak*, „it proceeds".

Commentary

Of the five currently known tablets and fragments concerned with Jupiter's scheme X.S$_1$, this is the best preserved one. However, compared to the other tablets its formulation is more terse. The name of Jupiter is not mentioned and the significance of several parameters, e.g. 0;12 and 0;9,30, is not indicated. Two close duplicates of Text A, BM 36801 and BM 41043, published as Nos. 21 and 22 in (*2*), clarify the meaning of Text A. They do mention the name of Jupiter and they qualify 0;12 as a value of its daily displacement (along the ecliptic), as can be seen in lines 1-3 of BM 36801:

Transliteration

1 MUL$_2$.BABBAR ME IGI 12 ZI-*šu*$_2$ ⸢EN⸣ [... A.RA$_2$]

2 1 ME DU-*ma* 10.45 TA 1 ME ⸢GI⸣ [...]

3 5.30 5.30 A.RA$_2$ 1 ME DU-⸢*ma*⸣ [5.30 ...]

Translation

1 Jupiter. The day when it appears its (daily) displacement is 0;12, until [... times]

2 1,0 days you multiply, it is 10;45. After completing 1,0 days [...]

3 0;5,30. You multiply 0;5,30 by 1,0 days, it is [5;30 ...]

This agreement between Text A and BM 36801 proves that Text A pertains to Jupiter and that the numbers 0;12 and 0;9,30 are values of its daily displacement. That this motion proceeds along the ecliptic and that the displacements are measured in degrees follows from a comparison with other tablets about planetary motion (*1, 2*).

<u>General remarks about Texts B-E</u>

Texts B-D and, as was argued, Text E, deal with the same trapezoid with short side 0;9,30, long side 0;12 and width 1,0, but they are not exact duplicates. That is, in each of the four texts the trapezoid procedure is formulated in slightly different words. Several phrases appear in more than one text, but not always in the same order. Text E deviates the most, being the only one that partly preserves the end of part II.

Text B

Text B is written on the fragment BM 34757, which measures 5.1 x 6.3 x 2.0-2.9 cm (Fig. S1). It arrived in the British Museum around 1879, having been excavated unscientifically in Iraq, most likely in Babylon (*16*). The fragment belongs to the left half of a tablet. Apart from the left edge, no other edges of the original tablet are preserved. The textual restorations imply that about 2 cm of clay are missing from the right side; hence the original width was about 7 cm. Text B occupies one side of BM 34757, which might equally be the obverse or the reverse. The other side partly preserves three more procedures about Jupiter. For an edition of the complete tablet see No. 38 in (*2*). The present edition incorporates several significant corrections to the previous one (*2*).

Transliteration

1 ⸢xx MUL$_2$⸣.BABBAR *ina šu-tam-ḫi-*⸢*ri*⸣ [12 SAG]

2 GAL-*tu*$_2$ 9.30 SAG TUR-*tu*$_2$ UŠ-*šu*$_2$ E[N.NAM]

3 10.45 *ana* UGU KI IGI TAB-*ma ina* 1-*šu* [xx]

4 DU$_3$.DU$_3$.BI *ša*$_2$ SAG.KI GU$_4$ SAG-⸢*su*⸣ [xx]

5 *ana muḫ-ḫi a-ḫa-miš*$_2$ TAB-*ma* 1/2-*šu*$_2$ GIŠ-⸢*ma*⸣ [A.RA$_2$ 1-*šu* DU-*ma*]

6 10.45 A.ŠA$_3$ 10.45 *ana* UGU KI [IGI TAB xx]

7 *ṣal-pi ina* KASKAL-*šu*$_2$ *ina* ŠA$_3$-*šu*$_2$ (*erasure*) SE$_3$.AM$_3$ ⸢*u*$_4$⸣-[*mu ša*$_2$]

8 *ana ṣal-pi* DIB-*iq tam-mar ki-ma-a* ⸢*mu*⸣-[*šam-ša*$_2$ x]

9 10.45 *šu-u*$_2$ *tam-mar ki* BAR *šu-u*$_2$ 12 ⸢SAG⸣ [GAL-*tu*$_2$ A.RA$_2$ 12 DU-*ma*]

10 2.24 9.30 SAG *ṭe-ri-tu*$_2$ A.RA$_2$ 9.3[0 DU-*ma* 1.30.15]

11 TA 2.24 ZI-*ma ša*$_2$ *re-ḫi* A.RA$_2$ KI [xxx]

12 ⸢*ša*$_2^?$ 10.4⸣5 DU-*ma* 26.52.30 ⸢9.30 xx⸣ [xx]

(unknown number of lines missing)

Translation

1 ... of Jupiter by squaring. [0;12 is]

2 the large [side], 0;9,30 is the small side, w[hat] is its station?

3 You add 10;45 to the position of appearance, and in sixty [days ...]

4 Its procedure: the [...] side(s) of the trapezoid

5 you add together, you compute half of it and [you multiply it by sixty days, it is]

6 10;45, the area; 10;45 [you add] to the position [of appearance. ...]

7 You place the crossing in its path, in its middle. The da[y when]
(or: You se[e] the crossing in its path, whereby it is halved. [The day when])

8 it passes the crossing, you see how long it is spen[ding ...]

9 (Concerning) this 10;45, you see when (or: how; that) it is halved: 0;12, the [large] side, [you multiply by 0;12, it is]

10 0;2,24. 0;9,30, the pinched side, [you multiply] by 0;9,30, [it is 0;1,30,15.]

11 You subtract it from 0;2,24, what remains you multiply by ... [...]

12 of$^{?}$ 10;45, it is 0;0,26,52,30. 0;9,30 ... [...]

(unknown number of lines missing)

Philological remarks

1) The first two signs are too strongly damaged to be identified. At the end of the line, traces compatible with the beginning of *ri* are visible. *šu-tam-ḫi-*⸢*ri*⸣: *šutamḫiru* is most likely a variant spelling of *šutamḫuru*, infinitive of the Št stem of *maḫāru*, which means „to square" in mathematical contexts (*4*). Apart from the present instance this verb is not attested in the astronomical corpus. Because several numbers are squared later on in the procedure, „squaring" might be the appropriate translation. However, since the phrase occurs at the beginning of the procedure it may also be interpreted as a more general reference to the geometrical method underlying the entire procedure.

2) UŠ-*šu*$_2$: in the present context this most likely represents *nēmettašu*, „its station", i.e. Jupiter's first station. The traces that remain of the last sign are compatible with E[N.NAM] = *mīnû*, „what", as proposed in (*1*).

3) 1-*šu* = *uššu,* „sixty": here the number 60 is written semi-phonetically and not in sexagesimal place value notation, perhaps in order to avoid a misinterpretation, since in that notation 1 and 60 would both be rendered as 1.

7) *ṣal-pi*: the technical term *ṣalpu* is tentatively translated as „crossing" (*2*). After ŠA$_3$-*šu*$_2$ one or two signs were partly erased, most likely on purpose by the scribe. The surviving traces suggest that *ṣal-pi*, „crossing", was previously written there. SE$_3$: in the present context, two readings of this logogram appear possible. First, a form of *mašālu*, „to be half; equal", most likely a stative of the G stem, *mašil*, „it is halved". The pseudo-Sumerian affirmative suffix AM$_3$ can be interpreted as a marker of this stative form. In this reading of SE$_3$ the preceding *ina libbi*(ŠA$_3$)-*šu*$_2$ is most suitably interpreted as the instrumental conjunction „whereby" (*20*). A second plausible option is to interpret SE$_3$ as a second person of the present tense of the G stem of *šakānu*, „to place", i.e. *tašakkan*, „you place", in which case *ina libbi*(ŠA$_3$)-*šu*$_2$ can be translated „within it; in its middle". Neither interpretation can be ruled out.

8) *tam-mar ki-ma-a* ⸢*mu*⸣-[*šam-ša*$_2$ x], „you see how long it is spend[ing ...]": as in line 9, the phrase following *tammar*, „you see", is best interpreted as an indirect question (*17-19*). For a discussion of *mušamšâ* see the commentary to Text D, line 6.

9) 10.45 *šū tammar kī mašlu*?(BAR) *šū,* „(Concerning) this 10;45: you see when it is halved". Since 10.45 *šū*, „this 10;45", is in the nominative case, it is not the object of the following *tammar*, „you see". Instead, the object of *tammar*, „you see", is the immediately following clause introduced by the conjunction *kī*, „when; how; that" (*17-19*). In the present context, the most suitable interpretation of this clause is an indirect question („when; how"). Alternatively it can be an object clause („that"), but they are usually introduced by *ša* (*18*). The subject of the clause is *šū*, „that; this; it", the predicate BAR, which most likely represents a form of *mašālu*, „to be half; equal". It is here assumed to represent *mašlu,* a subjunctive of the stative form of the G stem, „it is halved", but some other form of this verb would also be possible.

12) ⸢*ša*$_2$? 10.4⸣5, „of 10;45": the meaning of this incompletely preserved phrase is not fully clear. It presumably belongs to a descriptive term that qualifies the factor 0;30 (=1/2) by which the outcome of line 11 is multiplied here (see the commentary below).

Commentary

Text B preserves 12 nearly complete lines of a trapezoid procedure. Line 1 is probably the actual first line of the procedure. Lines 1-6 belong to part I; lines 7-12 to part II, which continued beyond line 12 for an unknown number of lines.

Part I: in lines 1-2 two measures of the trapezoid are preserved. The problem to be solved is formulated as a question for the station of Jupiter (line 2). In line 3 the number 10;45 is added to the position of Jupiter at its first appearance, i.e. $\lambda_{60}=\lambda_0+10;45^o$. The corresponding time interval of 60 days is also preserved. In the following lines this number, 10;45, is computed as the area of the trapezoid (lines 4-6). Its addition to Jupiter's position at the first appearance is then repeated, i.e. $\lambda_{60}=\lambda_0+10;45^o$ (line 6).

Part II begins with a statement concerning the „crossing" (line 7), which is said to be located in the middle of Jupiter's path. This is followed by a statement of the quantity that is asked for, namely the time in which Jupiter reaches the „crossing" (lines 7-8). The solution procedure is partly preserved in lines 9-12. The quantity asked for is here specified as the time in which Jupiter covers half the distance 10;45 (line 9). The solution procedure begins with a computation of v_0^2 (line 9) and of v_{60}^2 (line 10). The latter is subtracted from the former, $v_0^2–v_{60}^2$, which is multiplied by 0;30 (=1/2), leading to (0;2,24–0;1,30,15)/2 = 0;0,26,52,30 = u^2.

<u>Text C</u>

Text C is written on BM 34081+34622+34846+42816+45851+46135, which measures 22.6 x 13.1 cm. The six known fragments of this tablet arrived in the British Museum between 1878 and 1881, having been excavated unscientifically in Iraq, most likely in Babylon (*16*). The left and right edges of the original tablet are partly preserved, but nothing remains of the original upper and lower edges. On both sides the tablet is divided into three columns. It is inscribed with a compendium of at least 32 procedures for

Jupiter. Text C is the fifth procedure, which occupies lines 20-24 of column i on the obverse (Fig. S2). For an edition and photograph of the complete tablet see No. 18 in (*2*). The present edition incorporates several corrections to the previous one (*2*).

Transliteration

20 [xxxxxxx] 1-*en* SAG.KI GU$_4$ *ša*$_2$ 12 SAG

21 [GAL-*tu*$_2$ 9.30 SAG TUR-*tu*$_2$ xx] ⌜1-*šu*⌝ ME 10.45 A.ŠA$_3$-*šu*$_2$

22 [xxxxxxxx] ⌜ME⌝ 10.45 KI DU

23 [xxxxxxxxx] ⌜x⌝ *ṭe-ri-tu*$_2$

24 [xxxxxxxxxxx] ⌜x⌝

[unknown number of lines missing]

Translation

20 [...] one trapezoid of which the [large] side is 0;12,

21 [... the small side is 0;9,30 ... times] sixty days is 10;45, its area.

22 [... In sixty] days the position proceeds 10;45.

23 [...] pinched ...

24 [...] ...

[unknown number of lines missing]

Philological remarks

22) The new reading *qaqqaru*(KI) *illak*(DU), „the position proceeds“, replaces E$_{11}$, „you subtract“ (*2*).

Commentary

Text C preserves up to 9 final signs of the first 5 lines of a trapezoid procedure. In what remains of part I (lines 20-22), two measures of the trapezoid are provided (lines 20-21) and its area is obtained as 10;45 (line 21). In line 22 this is declared to be the distance by which the position (of Jupiter) proceeds, i.e. $\lambda_{60}=\lambda_0+10;45^o$. What remains of lines 23-24 is insufficient for identifying their meaning, but they probably belong to part II, which continued below line 24 for an unknown number of lines.

<u>Text D</u>

Text D is inscribed on the previously unpublished fragment BM 35915, which measures 3.5 x 4.0 x 0.8 cm (Fig. S3). It arrived in the British Museum around 1880, having been excavated unscientifically in Iraq, most likely in Babylon (*16*). No edges of the original tablet are preserved. Text D occupies one entire side of the fragment; the other side is destroyed.

Transliteration

[unknown number of lines missing]

1 [...] ⸢xxxx⸣ [...]

2 [... 10].45 *ana* UGU KI I[GI TAB-*ma* ...]

3 [...] ⸢*ki*⸣-*ma-a ana ṣal-pi* DIB [...]

4 [... 10.45 *šu*]-⸢*u*$_2$⸣ *tam-mar ki-ma* BAR-*šu*$_2$ [...]

5 [... 9.30] SAG.KI *ṭe-ri-tu*$_2$ A.⸢RA$_2$⸣ [9.30 DU-*ma* 1.30.15 ...]

6 [...] ⸢DU?⸣ *ki-ma mu-šam-ša*$_2$ ⸢x⸣ [...]

7 [...] ⸢1⸣.30.⸢1⸣5 *ana* UGU ⸢x⸣ [...]

8 [...] ⸢DU?⸣ *al-la* ⸢DIB?⸣ [...]

9 [...] ⸢x *ana*⸣ *ṣal-pi* [...]

[unknown number of lines missing]

Translation

[unknown number of lines missing]

1 [...] ... [...]

2 [... 10];45 [you add] to the position of appea[rance, and ...]

3 [...] How long does it pass to the crossing [...]

4 [... Concerning th]is [10;45], you see when (or: how; that) it is halved: [...]

5 [... You multiply 0;9,30], the pinched side, times [0;9,30, it is 0;1,30,15 ...]

6 [...] You multiply [... times ...] ... how long it is spending ... [...]

7 [... You add] 0;1,30,15 to ... [...]

8 [...] ... beyond ... [...]

9 [...] ... to the crossing [...]

[unknown number of lines missing]

Philological remarks

4) For this clause see Text B, line 9. BAR can be read with a form of the verb *mašālu*, „to halve; be equal", but it might also be read „half; 1/2", in which case one can translate „... you see how its half [...]".

6) *mu-šam-ša₂*: the most plausible interpretation is *mušamšâ*, a participle of *šumšû*, literally „to spend the night“, which presumably has the more general meaning „to spend time; stay“ here. See also the philological remarks to line 8 of Text B.

Commentary

Text D preserves portions of 9 lines of a trapezoid procedure. Lines 1-2 belong to part I; lines 3-9 to part II. Line 1 was most likely preceded by several more lines and part II continued below line 9 for an unknown number of lines. The name of a planet is not preserved, but a comparison with Texts A-C strongly suggests that Text D deals with Jupiter.

In what remains of part I, 10;45, Jupiter's total displacement after 60 days, is added to its position at first appearance, i.e. $\lambda_{60}=\lambda_0+10;45^\circ$ (line 2). The problem to be solved in part II, namely to compute the time in which a planet, most likely Jupiter, covers half the distance 10;45, is partly preserved in line 3. The solution procedure is introduced in line 4 and then executed. Line 5 preserves the computation of v_{60}^2. The computation of v_0^2=0;2,24 is missing, but a comparison with Text B (lines 9-10) suggests that it was written at the end of line 4. In line 7 v_{60}^2 is added to something, presumably v_0^2. Lines 8-9 are too damaged for an interpretation.

Text E

Text E is inscribed on the tablet BM 82824+99697+99742, which measures 8.3 x 5.3 x 1.2 cm. The three fragments arrived in the British Museum in 1883 and 1884, having been excavated unscientifically in Iraq, most likely in Babylon (*16*). The fragments do not include any portion of the original edges of the tablet. Only one side is inscribed; the other side is destroyed. Text E is the first preserved procedure (Fig. S4). It is followed by two more procedures, both concerned with Jupiter. The textual restorations in the latter imply that not much clay is missing from the left side and perhaps a few cm from the right side. For an edition and a photograph of the complete tablet see No. 40 in (*2*). The present edition incorporates numerous corrections to (*2*).

Transliteration

[unknown number of lines missing]

1 [...xxxxxxxxxx] ⸢xxxx⸣ [...]

2 [...] ⸢UŠ? xxx⸣ [xxxx] ⸢x UŠ 12?⸣ SAG.KI G[AL-*tu₄* ...]

3 [...] ⸢x⸣ SAG.KI TUR-*tu₄* ⸢E? x⸣ [xxx] ⸢30? *uz*⸣-*zab-bil* IGI 2.50? [...]

4 [...] TA U₄.MEŠ.AM₃ KI UD ⸢x *u₄-mu*⸣ *ša₂* KI ⸢*ṣal*⸣-*pi* DIB-*qa tam-mar* 3⸢2⸣ [...]

5 [...] *re-ḫi* 27? 28 KI ⸢U₄⸣-*ka* TAB-*ma u₄-me* ⸢KI *ṣal-pi*⸣ DIB-*qa tam-m*[*ar* ...]

6 [...] ⸢*ki*⸣-*i* TAG₄ 9.⸢30⸣ *ana* UGU 10.50 TAB-*ma* 1/2-*šu₂* GIŠ-*ma* A.RA₂ 32 ⸢U₄⸣.

[MEŠ DU-*ma* 5.25.20 ...]

7 [...] ⸢xx BU.AM$_3$⸣ 10 *tu*$^!$*-šaḫ-ḫi-zu* PI ⸢AN⸣ NU TUKU 50 *ana tar-ṣa* ⸢x⸣ [...]

8 [...] ⸢x *ša*$_2$⸣ U$_4$.MEŠ *ina* 1.3 ME E *ina* 2 UŠ *ina* 2.12 ⸢ŠU$_2$⸣ *ki-ṣir* AN ⸢x⸣ [...]

Translation

1 [...] ... [...]

2 [...] width? ... [...] ... the width, 0;12? the la[rge] side [...]

3 [...] ... the small side ... [...] 30? was carried ... [...]

4 [...] from the days ... You see the day when it passes the position of the crossing; 32 [... you subtract from 1,0 ...]

5 [...] there remains 27 (error for 28). You add 28 to your day, and you see the day when it passes the position of the crossing [...]

6 [...] what remains: you add 0;9,30 to 0;10,50, you compute half of it and you multiply it by 32 day[s, it is 5;25,20 ...]

7 [...] ... you let proceed ... nothing. 50. Opposite ... [...]

8 [...] ... of the days: in 1,3 rising to daylight, in 2,0 the station, in 2,12 the setting: the node of ... [...]

Philological remarks

1) Only illegible traces are visible.

2) ⸢x UŠ⸣, „..., the width": the term UŠ, „width", follows a damaged sign, most likely a number. Only a final vertical wedge of that sign remains visible, compatible with 1-*šu,* „sixty". ⸢12?⸣: only one vertical wedge is clearly visible after the 10, which would yield 11, but they are separated by an anomalously large empty space. However, in that space are visible faint traces of another vertical wedge (Fig. S4), which prompts a tentative reading 12, to be interpreted as 0;12. The traces that follow *pūtu*(SAG.KI), „side", are compatible with *rabītu*(GAL), „large". Hence this line may be part of a declaration of the measures of a trapezoid of „width" 60 (days) and „large side" 0;12.

3) The term *pūtu*(SAG.KI) *ṣeḫertu*(TUR-*tu*$_4$), „small side", should be preceded by the numerical value of that side. The partly preserved sign might be a 50 or a 30 followed by a separator (:). E: replaces the previous reading PI, but the meaning remains unclear. ⸢30 *uz*⸣*-zab-bil*: replaces the previous reading ⸢10 x⸣ ŠU$_2$ ZALAG$_2$ GIBIL (*2*). The form *uzzabbil* is a preterite tense of the Dt or Dtn stem, or a perfect tense of the D stem of *zabālu*, „to carry". This verb is thus far attested in mathematical contexts only in connection with brick computations (*21*), but the related verbs *wabālu*, „to carry", and *tabālu*, to „carry away", appear in mathematical texts with a meaning „to multiply" and „to subtract", respectively (*4, 21*). The present context is insufficient for determining the

intended meaning of *uzzabbil*. IGI? 2.50?: the 50 may also be read as 20 followed by a separator (:). The meaning of these signs is not clear.

4) [...] *ina*/*ultu*(TA) *ūmē*(U_4.MEŠ.AM_3) KI UD ⸢x⸣ [x], „[...] from the days ...": the meaning of this fragmentary phrase is unclear. KI: most likely to be read *itti*, „with", or *qaqqaru*, „position", but no plausible interpretation could be found. UD ⸢x⸣: the sign following UD begins with a vertical wedge. One might read *tam-mar*, „you see", but this does not result in a meaningful sentence, so the correct reading remains unclear. We are on firm ground from ⸢*u_4-mu*⸣, „day", onwards, because this phrase clearly belongs to part II of the trapezoid procedure. 3⸢2⸣: only the left wedge of the 2 is preserved before the break.

5) ⸢U_4⸣-*ka*, „your day": replaces the previous reading ⸢x⸣-*ka* (*2*). *etēqa*(DIB-*qa*). „to pass": replaces the previous reading HAB$^?$ UD (*2*).

7) This line contains a procedure for Jupiter that turns out to be distinct from the trapezoid procedure (see the commentary). ME E, „rising to daylight", is the synodic phenomenon of acronychal rising; UŠ, „station", here denotes the second station, and $ŠU_2$, „setting", is the last appearance (*2*).

8) This line also contains a procedure that is distinct from the trapezoid procedure (see the commentary).

Commentary

Lines 1-3 deal with a trapezoidal figure but they are difficult to interpret due to their bad state of preservation and the presence of signs and words for which no suitable translation could be established, in spite of several improved readings. The end of line 2 and the beginning of line 3 contain a declaration of the measures of the trapezoid that presumably belongs to part I of the trapezoid procedure. Lines 4-6 are now understood for the first time as belonging to part II of the trapezoid procedure. However, they do not constitute its beginning, which implies that some portion of lines 2-3 must also belong to part II. Even though a horizontal ruling is not visible below lines 6 or 7, but below line 8, it is now clear that the trapezoid procedure ends either at the end of line 6, or in line 7 (see below).

Line 5 begins with the phrase „there remains". In Late Babylonian astronomical and mathematical texts this introduces the outcome of a preceding operation (*2*). This outcome is nearly always repeated when it is passed on to an immediately following computation (*2*). Here the result is passed on as 28, but the number preceding it looks like 27. Hence this 27 is very likely a damaged 28 or a scribal error for 28. The phrase „there remains" occurs almost exclusively after subtractions (*2*), which strongly suggests that the 28 was obtained by subtracting 32 days, the number partly preserved at the end of line 4, from 60 days, which can be restored in the following gap. In line 5 the 28 days are to be added to „your day", resulting in the „day when it (= Jupiter) passes the position of the crossing“, which confirms that 28 days is a value of t_c, in agreement with the interpretation of the trapezoid algorithm.

In line 6 the area of the second partial trapezoid is computed, very likely in order to verify the partition. The correctness of this interpretation is supported by an Old Babylonian mathematical tablet, UET 5, 858, on which the partition of a trapezoid of different dimensions is followed by exactly the same type of verification (*5, 11*).

Line 7 is very difficult to interpret. It is provisionally assumed here that the trapezoid procedure ended in line 6, but some initial part of line 7 may constitute the end of the trapezoid procedure. In line 8 we have definitely left the trapezoid procedure, since it deals with a new topic, namely the duration of various intervals between Jupiter's synodic phenomena measured in days. 1,3 (=63) days is close to 60 days, attested as the time between the first station and the acronychal rising of Jupiter (*2*). 2,0 (=120) days is an attested value of the time between the first station and the second station (*2*). 2,12 (=132) days is an attested value of the time between second station and last appearance (*2*).

Fig. S1.
Photograph of Text B (lines 1-12). Reproduced from (*2*).

Fig. S2
Photograph of Text C (lines 20-24).

Fig. S3.
Photograph of Text D (lines 1-9).

Fig. S4
Photograph of Text E (lines 1-8).

References and Notes

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