Rostovtzeff Lectures 2018

The Sky over Ancient Iraq:
Babylonian Astronomy in Context

3 Algorithms, Tables and Figures: New Insights into Babylonian Mathematical Astronomy

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April 4 2018

1 mathematical astronomy – sources

2 tabular texts, arithmetic methods (Jupiter)

3 geometric methods (Jupiter)

4 historical perspective

747 BCE *Nabonassar*

Neo Assyrian era

Assurbanipal

625

Neo Babylonian era

Nebukadnezzar

538 *Cyrus*

Persian era

Xerxes

331 *Alexander the Great*

Seleucid era

141 BCE

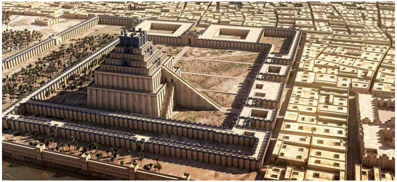
Parthian era

1 Babylonian mathematical astronomy – sources

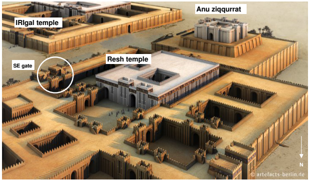
basic facts about the text corpus

- ca. 350 tables with computed positions & other phenomena of moon (50%), five planets (49%), sun (1%)
- ca. 110 procedure texts with instructions, mainly for computing these tables
- written ca. 380–50 BCE
- ca. 370 from Babylon, ca. 90 from Uruk
- main collections: London; Istanbul; Chicago; Berlin, Paris, Baghdad
- editions: O. Neugebauer, 1955, *Astronomical Cuneiform Texts*; M. Ossendrijver, 2012, *Babylonian Mathematical Astronomy: Procedure Texts*

British Museum



Babylon



Uruk

institutional setting: Esagila temple (Babylon) and Rēš temple (Uruk) *lecture 4*

applications: calendar; astrology *lecture 4*

2 tabular texts: planets

first stations of Jupiter

Seleucid Era 113–173 (199–139 BCE)

- AO 6476 (Paris) + U 104 (Istanbul)
- excavated in Uruk ca. 1912
- from Rēš temple, written SE 118 (194 BCE)
- edition: O. Neugebauer, 1955, *ACT* No. 600
- algorithm: system A

160	XII_2	48;5,10	II	10;29	28;6	Capricorn
⌜1⌝62		48;5,10	II	28;34,10	4;6	Pisces
⌜1⌝63		48;5,10	IV	16;39,20	10;6	Aries
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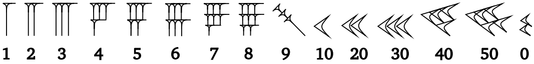
⌜1⌝70	VI_2	42;5,10	XI	23;44,30	22;35	Scorpio
172	XII_2	46;36,10	I	10;20,40	27;6	Sagittarius
173		48;5,10	I	28;25,50	3;6	Capricorn

First Station of Jupiter.

numbers: sexagesimal place value notation

- base number 60
- invented ca. 2000 BCE (for computation only)
- 0s at beginning or end of number not written

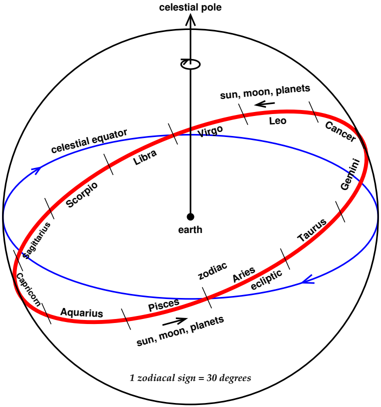


examples:

1,0 (sexagesimal) = 60 (decimal)

10;45 (sexagesimal) = 10 + 45/60 = 10.75 (decimal)

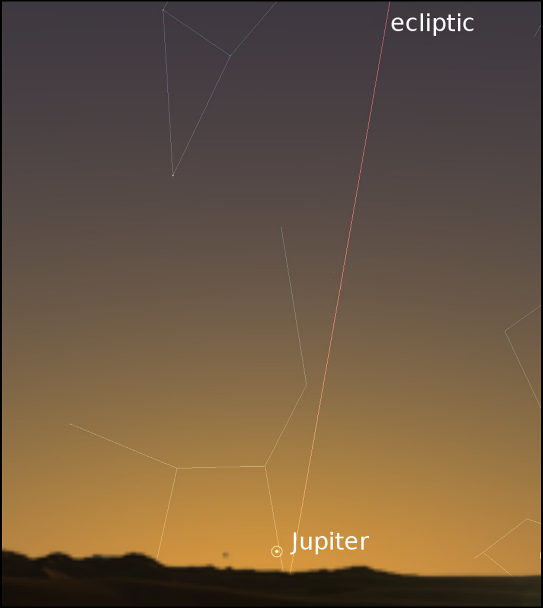
0;10,45 (sexagesimal) = $10/60 + 45/60^2$ (decimal)



zodiac:

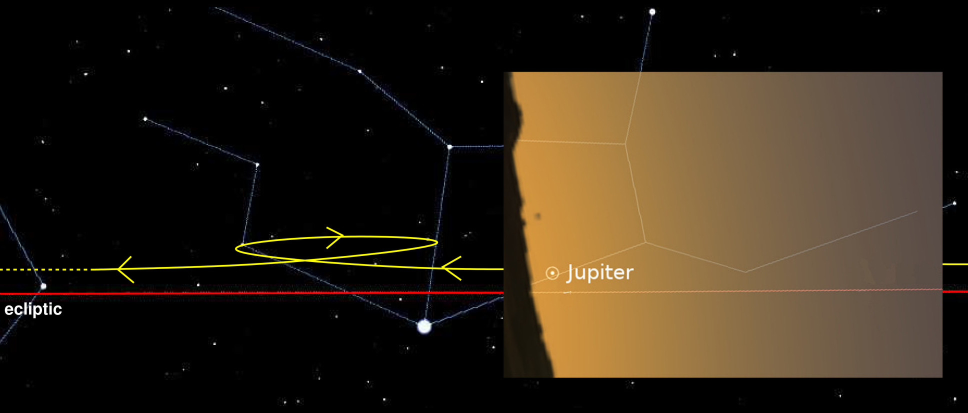
- ecliptic divided into 12 named signs of 30 degrees (UŠ)
- invented ca. 410 BCE
- positions indicated by zodiacal sign and degrees within it

2 Jupiter's synodic cycle



first appearance (just before sunrise, eastern horizon)

2 Jupiter's synodic cycle



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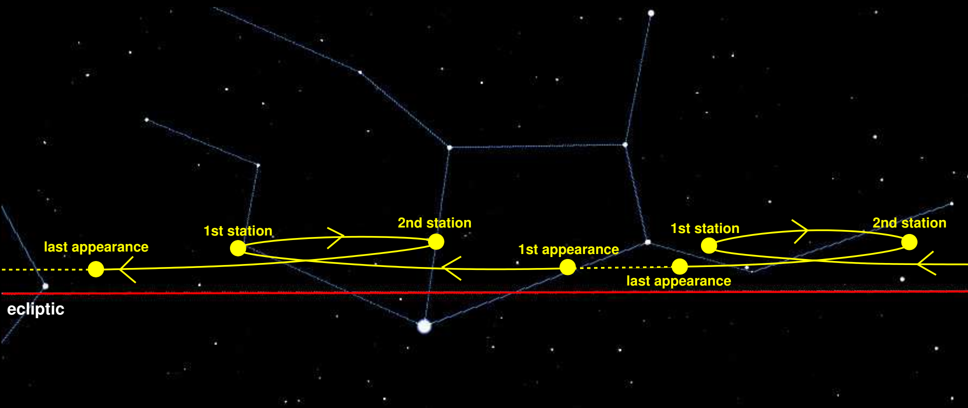

2 tabular texts: planets

first stations of Jupiter, SE 113–173 (199–139 BCE)

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First Station of Jupiter.

- tablet: AO 6476 (Louvre) +U 104 (Istanbul)
- from Uruk, Rēš temple, SE 118 (194 BCE)
- edition: O. Neugebauer, 1955, *ACT* No. 600
- algorithm: Jupiter system A

corresponding procedure:

(First) appearance to (first) appearance.

From 25 Gemini until 30 Scorpion you add 30.

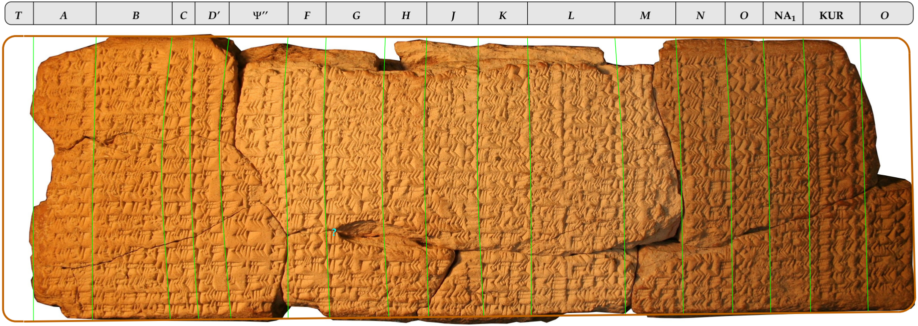
From 30 Scorpion until [25] Gemini you add 36.

- tablet: BM 34081+, from Babylon, 350–50 BCE
- compendium of Jupiter procedures
- editions: Ossendrijver 2012 No. 18; Neugebauer 1955 No. 813

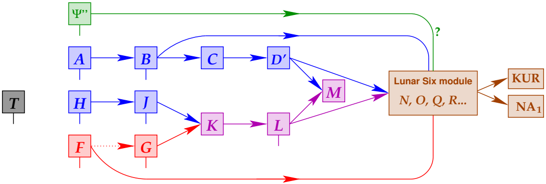
2 tabular texts: moon

New Moons (conjunctions moon – sun) for SE 208–210 (102/1–100/99 BCE)

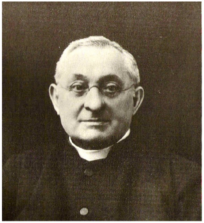
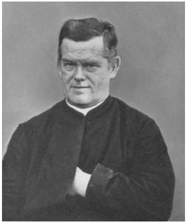
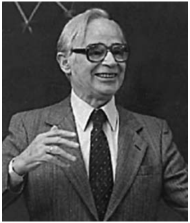
BM 34580+ (*ACT* 122), written 103/2 BCE, Babylon



A	monthly displacement along zodiac
B	zodiacal position
C	duration of daylight
D′	duration of half night
Ψ''	'eclipse magnitude'
F	Moon's daily displacement along zodiac
G	time between successive lunations − 29 days
H	monthly difference of *J*
J	zodiacal correction to *G*
K	*G* + *J* = monthly difference of *L*
L, *M*	time of lunation
O	≈ elongation between Moon and Sun
NA_1	time between sunset and first visible moonset
KUR	time between last visible moonrise and sunrise



2 past research on Babylonian mathematical astronomy

				
Joseph Epping 1835–1894 mathematician	Johann Strassmaier 1846–1920 oriental scholar	Franz Kugler 1862–1929 chemist	Otto Neugebauer 1899–1990 mathematician	Bartel van der Waerden 1902–1996 mathematician

1881	Epping, Strassmaier: *On the Decipherment of the Astronomical Tablets of the Chaldeans*
1900–1924	Kugler: *Babylonian Lunar Computation*; *Astronomy and Astral Religion in Babel*
1955	Neugebauer: *Astronomical Cuneiform Texts (ACT)*
1965/1974	van der Waerden: *Science Awakening II. The Birth of Astronomy*
1975	Neugebauer: *A History of Ancient Mathematical Astronomy (HAMA)*

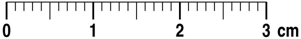
established "arithmetic view" on Babylonian astronomy (O. Neugebauer in *HAMA*):

> *From the cuneiform texts we learned that ephemerides had been computed exclusively by means of intricate difference sequences which, often by the superposition of several numerical columns, gave step by step the desired coordinates of the celestial bodies — all this with no attempt of a geometric representation, which seems to us so necessary for the development of any theory of natural phenomena. It is a historical insight of great significance that the earliest existing mathematical astronomy was governed by numerical techniques, not by geometric considerations (...).*

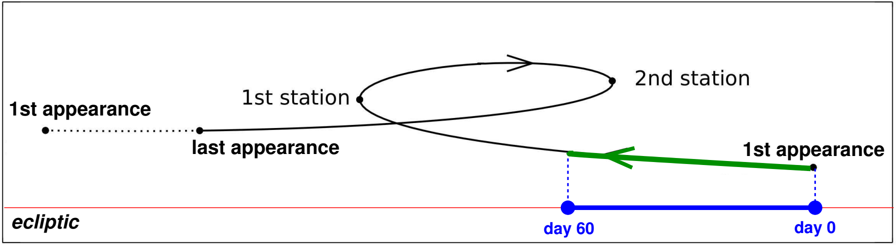
3a new numerical scheme for Jupiter's motion

- BM 40054 (British Museum), from Babylon, ca. 300–50 BCE
- nearly complete description of Jupiter's motion along ecliptic during 1 cycle
- special feature: intervals with linearly changing "velocity" (degrees/day)
- 8 duplicates, from Babylon

Text A (BM 40054)



M. Ossendrijver, 2017, "New results on a Babylonian scheme for Jupiter's motion along the zodiac", *Journal of Near Eastern Studies* 76, 231–247



Jupiter's motion from 1st appearance to 1st appearance

3a numerical scheme for Jupiter's daily motion

beginning of Text A and duplicates:

The White Star (=Jupiter): on the day of its (1st) appearance
its displacement is 0;12, until 1,0 (=60) days 0;9,30.
You add 0;12 and 0;9,30, it is 0;21,30.
You multiply it by 0;30 (=1/2), it is 0;10,45.
You multiply it by 1,0 (=60) days, it is 10;45.

interpretation: arithmetic computation of total distance

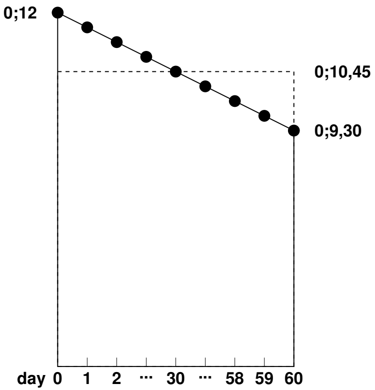
„displacements" 0;12 and 0;9,30: velocities in degrees/day

$(0;12 + 0;9,30)/2 = 0;10,45$ = mean velocity

$0;10,45 \times 1,0 = 10;45$ = total distance covered in 60 days

modern formula: total distance $D = 60 \cdot (v_0 + v_{60})/2$

implies following velocity scheme, as modern graph:



3b geometric computation of same distance

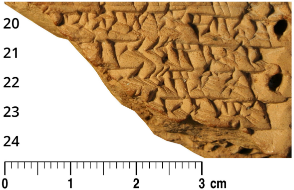
Text B

BM 34757

Text D

BM 35915

Text C



BM 34081+34622+34846+42816+45851+46135

Text E

BM 82824+99697+99742

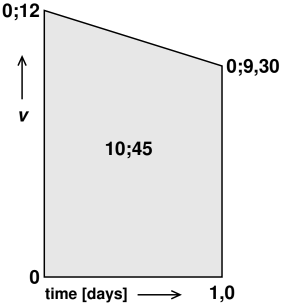
from Babylon, ca. 350–50 BCE (courtesy Trustees of the British Museum)

3b geometric computation of same distance

Text B (BM 34757), part 1:

[...] ... of the White Star ...: [0;12] is the large side, 0;9,30 is the small side, what is its ...?

Its procedure: the sides of the trapezoid you add, you compute half of it and you [multiply it by sixty (days)], it is 10;45, the area. You add 10;45 to the position [of the (first) appearance, and in sixty days ...]



Interpretation:

10;45 = area of trapezoid = distance covered by Jupiter after 60 days

distance covered by decelerating body = area of graph of "velocity" against time

3b geometric computation of same distance

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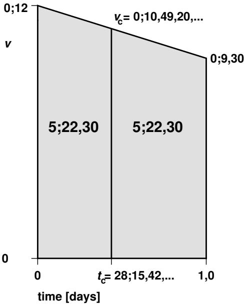

Interpretation:

10;45 = area of trapezoid = distance covered by Jupiter after 60 days

distance covered by decelerating body = area of graph of "velocity" against time

part 2: bisection of trapezoid

to answer question:

when does Jupiter reach half the distance of 10;45°?

4 historical perspective

Babylonian astronomers supposedly only used arithmetic methods, no geometric methods

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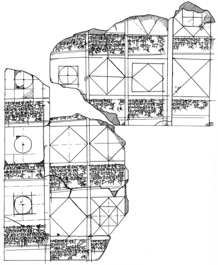
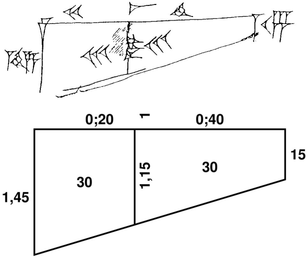
First Station of Jupiter.

they also applied geometric methods from Old Babylonian mathematics (1800–1600 BCE):



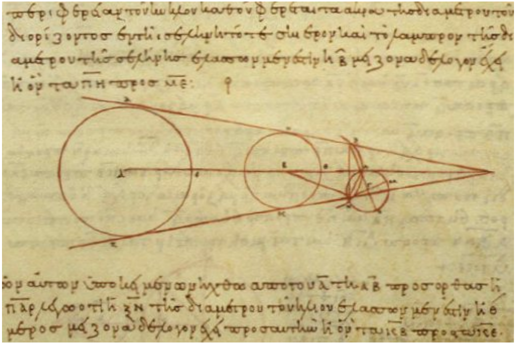
geometric problem text (1800–1600 BCE)



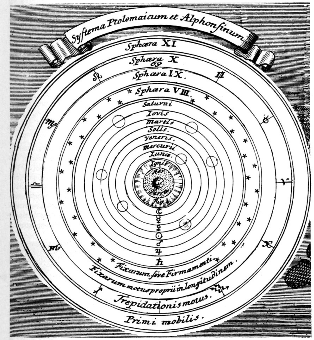
bisection of a trapezoid (1800–1600 BCE)

4 historical perspective

in antiquity only Greek astronomers were assumed to have used geometric methods



treatise by Aristarchos of Samos (2nd c. BC)



world system of Claudius Ptolemy (2nd c. AD)

computation of distance from area under velocity curve supposedly invented in Europe in 14th c. AD

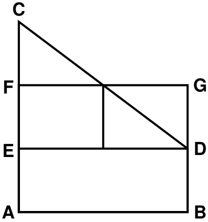
left: Merton College, center of "Oxford Calculators" (1350–1400)

"Mean Speed Theorem" for uniformly de- or accelerating bodies:

"covered distance = mean velocity × time"

$S = \frac{1}{2}(v_0 + v_1) \cdot t$

Nicole of Oresme (1320–1382, Paris)



Oresme's geometric proof

from: "Treatise on the configurations and the qualities of motions" (*Tractatus de configurationibus qualitatum et motuum*),

Chapter III, §vii, "On the measure of difform qualities and velocities" (*De mensura qualitatum et velocitatum difformium*)

Science 351, 29 Januar 2016

Links:

http://science.sciencemag.org/content/351/6272/482.full

http://science.sciencemag.org/content/351/6272/482/suppl/DC1

more detailed analysis and Old Babylonian origins:

M. Ossendrijver, 2018, "Bisecting the Trapezoid: Tracing the Origins of a Babylonian Computation of Jupiter's Motion", *Archive History of Exact Sciences* 72

translation and analysis of tablets with numerical scheme:

M. Ossendrijver, "New results on a Babylonian scheme for Jupiter's motion along the zodiac", *Journal of Near Eastern Studies*, 2017