

Complex analysis
Summer semester 2015

Exercise Sheet 5

Hand-in date: 12:00, Friday 15th May.

Exercise 1. (Inequality for complex-valued integrals along a real interval). Let $a < b$ be real numbers and $f : [a, b] \rightarrow \mathbb{C}$ be a continuous function. By writing the integral

$$\int_a^b f(t) dt$$

in polar coordinates, show that

$$\left| \int_a^b f(t) dt \right| = \int_a^b \operatorname{Re}(\exp(-i\theta) f(t)) dt$$

for some real number θ . Deduce that

$$\left| \int_a^b f(t) dt \right| \leq \int_a^b |f(t)| dt.$$

Exercise 2.

- a) Does the imaginary part function $\mathbb{C} \rightarrow \mathbb{C}$ given by $z \mapsto \operatorname{Im}z$ have a primitive?
- b) Calculate the integral of the following functions on a piecewise smooth curve from $1 + i$ to i
 - i) $z \exp(iz^2)$
 - ii) $\cos((1 + i)z)$

Exercise 3. Does the function $f : \mathbb{C}_- \rightarrow \mathbb{C}$ given by $z \mapsto 1/z$ have a primitive? Show that, for $z \in \mathbb{C}_-$, we have

$$\operatorname{Log}(z) = \int_{\gamma_z} \frac{1}{w} dw,$$

where we integrate along a straight line path γ_z from 1 to z .