Freie Universität Berlin
Institut für Mathematik
Stand: 14. Februar 2025

WiSe24/25
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Algebra I – Homework 16 – Bonus Sheet

Will be graded only for those who achieved **between 26 and 31 points** in the first 15 sheets.

Deadline: **20:00 on Friday 21.02.2025.** (Uploads are still possible until Sunday 23.02 at 23:55)

Submission: individually, on Whiteboard as `LASTname_A1_H16.pdf`

Full written proofs are required in support of your answers.

Problem 1. **2 bonus points**

Give an example of a ring R, $R-$modules M', M, N together with an injective R-linear map $M' \hookrightarrow M$, such that

$$N \otimes_R M' \neq 0 \quad \text{and} \quad N \otimes_R M = 0.$$

Problem 2. **2 bonus points**

A line in the affine space $\mathbb{A}^3_{\mathbb{C}}$ is an affine algebraic set defined by two linearly independent degree one equations. Alternatively, any line can be represented in parametric form as $\{(a,b,c) + t(u,v,w) \ : \ t \in \mathbb{C}\}$ where $(a,b,c),(u,v,w) \in \mathbb{C}^3$ and $(u,v,w) \neq (0,0,0)$.

Prove that the surcface $X = V(x^3 + y^3 + z^3 - 1) \subseteq A^3_{\mathbb{C}}$ contains at least three distinct lines.

Problem 3. **2 bonus points**

1. Are the graded rings $\mathbb{C}[x,y]/(x^2 + y^2)$ and $\mathbb{C}[x,y]/(xy)$ isomorphic?
2. What about the graded rings $\mathbb{C}[x,y]/(x^2 + y^2, x^2)$ and $\mathbb{C}[x,y]/(xy, x^2)$?

Total: 6 bonus points