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The Productivity of Blanks: On the Mathematical Zero and the Vanishing Point in Central Perspective. Remarks on the Convergences between Science and Art in the Early Modern Period

1. Instruments as Symbol-Technical Hybrids

We usually associate 'instruments' with man-made products such as tools, appliances, machines, or devices. Ranging from the screwdriver to the piano, up to the telescope and computer, we create objects which are then put to use to achieve particular goals. Speaking concretely, such an instrumentality makes up the core of our technical relation to the world. This rests on a further conviction: if the technical is fundamentally related to the instrumental, then the symbolic, that is, the use of signs, is not instrumental but should be understood as oriented towards understanding and interpretation. The materiality and sensual thing-ness, which is also part of the sign, is not then articulated in the termini of opaque objecthood, but on the foil of a transparent, transitory mediality. Here our understanding of instrumentality seems to establish a dividing line: the technical relates to the symbolic as production to representation (*Herstellen* to *Darstellen*), construction to interpretation, and as instrument to medium. Technical and symbolic procedures embody, therefore, two distinct domains of human poiesis, each with its own procedural and developmental logics.

Nevertheless, there are quite a few artefacts that block this schema of a disjunctive sorting into the technical or semiotic. Phonetic writing is a mechanism for setting down oral language – but is writing a technical or symbolic system? The decimal position system together with calculative algorithms allows a human calculator to 'mechanically' solve all the tasks of elementary arithmetic – so is the decimal system a calculating instrument or a numerical language? Is software, for example a

computer program that transforms a physical device into an Internet portal, now a machine or a text? The list of questions could easily go on.

Particularly in the field of cultural techniques, we stumble on such mixed forms that we want to call 'symbolic-technical hybrids.' Here, 'hybrid' relates to a combination of attributes originally belonging to distinct classes or fields of objects, which, however, cannot be distinguished from their 'mixed being'; rather, they exist simultaneously next to each other, and in this way remain sustained in their heterogeneity. The hybrid does not follow the logic of 'either-or'[1] but of 'both-and.' To come back to our examples, phonetic writing, the decimal system and its calculation rules, a computer program, are all in fact both: technology *and* language, instrument *and* medium. Seen from the viewpoint of such hybrids, 'technology' and 'symbol' become purifying stylizations, pole and borderline-case, of a scale (still) only grasped as a concept-grid, whose spectrum delivers what in the world we only encounter, in reality, as mixed relationships.

Our supposition, and also our hypothesis, is *that lasting shifts or even 'leaps' of a cultural dynamic touch on a – in cultural-technical terms – definable 'instrumentality' of just such symbolic-technical mixed forms*. This hypothesis concerns the foil on which convergences between instrumental innovations in science and art at the beginning of the modern period can emerge.

2. First an Anecdote

Let's start with a short story that Giorgio Vasari tells about Giotto.[2] The pope wants evidence of the art of Giotto di Bondone, a contemporary of Dante. When a courtier asks Giotto for a sample of his skill, Giotto takes a brush dipped in paint, and using his arm pressed to his side as a compass, paints a circle whose perfection causes much amazement. He passes the drawing on to the courtier, who is unsettled by the meagerness of the sample. The pope however understands and recognizes Giotto's genius.

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1 On the hybrid in Bakhtin's literary theory, cf. Irmela Schneider. "Von der Vielsprachigkeit zur 'Kunst der Hybridation.' Diskurse des Hybriden." *Hybridkultur. Medien, Netze, Künste*. Ed. idem and Christian W. Thomsen. Cologne: Wienand, 1997. 20.

2 Giorgio Vasari. *The Lives of the Artists*. Trans. Julia Conway Bondanella and Peter Bondanella. Oxford: Oxford University Press, 1998. 15-36. Concerning this anecdote, cf. Reinhard Steiner. "Malerei als Spekulation." *Digitaler Schein. Ästhetik der elektronischen Medien*. Ed. Florian Rötzer. Frankfurt a.M.: Suhrkamp, 1991. 435f.

What is revealing in this anecdote, which crystallizes a rupture in the fine arts into a legend? For a start, in relation to painting, drawing is revaluated. Moreover, the skill of the artist is demonstrated by the manner in which he is able to use his body as an instrument. Using his arm to imitate a compass, Giotto links *ars* and *techné*, painting and construction in the creative act. But what is particularly significant is that the painted image no longer follows the model of nature, i.e. a depiction, but sketches a mathematical object. Circles, in a literal sense, do not appear in nature – they are mathematical constructs. Plato was right when he insisted that the circle is not an element of the real world, but belongs to the world of the intellect,[3] since the circle drawn as a geometric figure is the sensible embodiment of a concept, a formula, and consequently a theoretical and abstract object. Therefore the painter's skill was directed towards a subject whose 'nature' it is to embody an object which can be constructed using calculations.

3. Calculability as a Symptom of Modern Art and Science, the Visualization of the Invisible, and the 'Discovery of the Subject' as the Origin of Numerical and Visual Space

When we speak of a convergence in the development of art and science in the early modern period, it is calculability that declines the mutual measure of both. The invention and dissemination of linear perspective on the one hand, and the language of formulas on the other, can both be understood as a symbolic form that seals a sublime connection.[4] Central perspective rationalizes our visuality; calculations rationalize our language. The construction of central-perspectival space sets out to objectify the phenomenon of seeing. The laws of perception are transformed into methodical and calculable rules of symbolic representation. The construction of calculations attempts to make operative the phenomenon of speaking, with the invention of a formal writing – this avoids vagaries in the use of natural languages and enables those who control the artificial alphabet of a calculation and its rules to identify errors in the representation as errors of calculation, and therefore to avoid them.

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3 Cf. Sybille Krämer. *Berechenbare Vernunft. Kalkül und Rationalismus im 17. Jahrhundert*. Berlin and New York: de Gruyter, 1991. 53ff.

4 Cf. Sybille Krämer. "Die Rationalisierung der Visualität und die Visualisierung der Ratio. Zentralperspektive und Kalkül als Kulturtechniken des 'geistigen Auges.'" *Bühnen des Wissens. Interferenzen zwischen Wissenschaft und Kunst*. Ed. Helmar Schramm et al. Berlin: Dahlem University Press, 2003. 50-67.

The 'perception of something' is made operative with the help of central perspective, just as 'speaking about something' is with the help of calculations. Consequently, what in artistic and scientific representation is made visible becomes in turn a mathematizable construction, and therefore a theoretical entity.

As soon as we describe an object as a 'theoretical entity,' it belongs – should we use a traditional Greek distinction – to the realm of 'logos' and not to that of 'aisthesis.' The 'logoi' themselves are 'anaisthetic,' that is, invisible. Only accessible to thought, they belong – at least in the traditional sense – to an ideal domain, which is outside space and time. If however central-perspectival and calculative representations make theoretical entities physically present, then both cultural techniques can also be viewed as techniques for visualizing the invisible. And that is the point towards which we want to direct our attention here. The arts are normally associated with the sensual and the concrete, while the sciences are associated with what can be formalized, quantified, and with abstraction. Our assumption, however, of a convergence between art and science is rooted in the conviction that calculations, in which the cognitive entities appear in the two-dimensionality of formulas, also makes use of the potential of a visualizing of the non-concrete and invisible.

The interplay of invisibility and visualization, belonging to the cultural techniques of central perspective and calculation, now corresponds to an interrelationship between the symbolic and the technical, since it is the (symbolic) act of the representation of an invisibility that can also be reconstructed as a (technical) act of the production of the represented. Now there is a phenomenon which is especially suited to demonstrate this connection of the sensualization of a non-sensual thing, which is only constituted at all as a result of this visualization, and that is the use of the figure '0' in the context of arithmetical calculations, in whose wake the cipher zero first comes to 'life' as a mathematical number; and it is also the use of the vanishing point, the 'visual zero'[5] in linear perspective, in relation to which the observer's point of view is promoted to the constructing principle of the picture. What unites both of these forms – at least this is our supposition – is that *the world of numbers as well as the world of images are structured such that, from this point of view, an epistemological subject can be made imaginable as the origin of numerical space as well as visual space*. Before we turn to the reasons for this supposition, a brief interjection seems in order.

5 Brian Rotman. *Signifying Nothing*. Basingstoke: McMillan, 1987. 19.

4. Interjection: Is the New Scientific Research a Contribution to the Performative 'avant la lettre'?

The idea that, with signs, something can not only be represented but also produced is a core idea of the concept of performativity. In his pioneering work *How To Do Things With Words*, John L. Austin revealed that there are verbal expressions which concurrently also carry out what they say:[6] a ship's blessing, a judge's ruling, the 'I do' of a marriage ceremony, testaments, but also promises, requests, orders. With the discovery of the performativity of linguistic expressions, Austin demonstrated a quasi-instrumental dimension in our speech: speech-acts create – at least under certain conditions – social facts. With their world-changing and world-forming power, verbal operations hardly come second to technical operations.

This 'instrumental revaluation' of our speech by speech-act theory subsequently effected linguistics, philosophy, art, and cultural theory. On the threshold of the twenty-first century, a 'performative turn' is looming.[7] Yet the theory and history of science (still) hardly refer to the performative, although the basic role of language as a means of presentation in science has become a commonplace. The fact that the linguistic aspect has fallen into the shadows has an understandable cause, since one of the most far-reaching changes of direction in scientific research intended that the notion of 'science' should not be reduced to the creation and verification of the theoretical forms of knowledge, that is, the propositional, and therefore to what is verbally explicable. As a result, non-verbal phenomena shifted into the focus of scientific research, such as the semiotically or technically supported practices in laboratories, the interaction of visuality and knowledge, image and text, the knowledge implicit in the use of scientific objects, the hybridization of things, symbols, and technologies in the day-to-day practice of the scientists. As much as we can discover in these approaches a reflection of the performative in science 'avant la lettre,' its determining feature remains a turning to the non-verbal aspect of the scientific process.

It is therefore time to test what insights are disclosed as soon as the idea of linguistic performativity is removed from its original domain, in colloquial language, and made fruitful for the observation of the role of scientific languages. The operative writing of calculations, i.e. formula-

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6 John L. Austin. *How to Do Things with Words*. Cambridge: Harvard University Press, 2005.

7 Uwe Wirth, ed. *Performanz. Zwischen Sprachphilosophie und Kulturwissenschaften*. Frankfurt a.M.: Suhrkamp, 2002.

language, is one of the most lasting forms of linguistic expression in the sciences. Where does it lead, if we now ask about the performativity of calculation, i.e. formal languages? Here we have come back to our theme.

5. On the Performativity of Calculation: When the Description Becomes the Execution of the Described

From a performative perspective, we need to clarify how to understand the fact that calculated expressions represent something, and also give rise to the represented.

To begin with, such a thesis finds itself in opposition to a readiness, which is common in the cultural sciences and inspired by debates in scientific research, to interpret the empirically operating and formalizing sciences not only as realistic, but also as instrumental: 'instrumental' in the sense that the statements of these sciences should not be interpreted as the depiction of an existing reality, but rather as the result of the interaction of people with the object-area to be investigated, so that it is legitimate to say that the things these sciences deal with are constructs arising as a result of scientific practices. Realism and instrumentalism contradict each other in relation to the question of whether theoretical entities actually exist (realism) or do not exist (instrumentalism). If we now approach this question in the context of a performative perspective, then we hope to be able to be in agreement with realism, in that the objects denoted by correct calculation-expressions actually exist, but that these objects are to be interpreted as the referents of calculations; these then – as instrumentalism supposes – do not exist independently of the symbolic mathematical practices. Here a revealing analogy for the question of the performative arises: just as performativity can create 'social facts' from (colloquial) speech-acts, that is, 'objects' whose existence is rooted in their social acknowledgement,[8] the performativity of calculation creates epistemological facts, whose existence equally depends on their being acknowledged.

Let us therefore turn to the symbolic act of formalizing. So-called formal languages are in reality writings: graphic systems sui generis, which then, in accordance with various colloquial languages, can also be uttered. The writing-character of 'formal language' is not a marginal characteristic but an essential one. Normally 'writing' is associated

8 Cf. John Searle. *The Construction of Social Reality*. New York: Free Press, 1995.

with written-down oral language. Here however, a more broadly applied notion of writing is called for which does not follow a phonetic writing-conception but views writing as a kind of hybrid-construction between language *and* image. In this way a phenomenon arises which can be too readily overlooked and forgotten in an observation reduced to the discursiveness of writing – 'Schrift*bild*lichkeit' (a hybrid of writing and image).[9] Unlike oral speech, which is dependent on the sequential nature of time, writing works with spatial configurations that make use of the two dimensionality of the surface. This spatiality opens up a kind of 'in-between-spatiality.' *Schriftbilder* are discrete, they work with gaps and empty zones, because of which letters first of all become unambigous, that is, disjunctively individualizable; no third sign can be placed between two signs. This exclusion of continuum on the part of the medium is the condition for the fact that we consider the represented in the medium as a system put together from identifiable elements. In the course of such a syntactically organized systematicity of the medium of representation, the represented takes on *nolens volens* systematic features itself. The visualization in the structure-image of writing subjugates what is visualized to the grid of a systematic construction principle. And we will see later, taking the numerical calculations of the decimal position system as an example, how important this transformation of numbers into a number-*system*, by means of notation in the form of a calculation, is; particularly when it is concerned with clarifying the meaning of zero.

If we also want to understand the calculated representation as the production of the represented, then at best we have so far stumbled on one *transformation*. A transformation that consists in the fact that the visualizing representation of contents in the disjunctive *Schriftbild* (writing-image) of calculation reveals this content precisely in its systematicity. If we speak of 'production' here, then this only relates, more precisely, to a commutation.

Hence we must take a further step; and only with this step do we stumble on a phenomenon that is revealed, exclusively, in the writing of calculations. Our previous considerations relate to the implicit structure-'producing' iconicity in calculation; now we want to turn to the operativity that is part of calculation. A calculus hides a double function: it is both language and technology, medium and instrument. In as

9 Cf. Sybille Krämer. "'Schriftbildlichkeit' oder: Über eine (fast) vergessene Dimension der Schrift." *Bild, Schrift, Zahl*. Ed. idem and Horst Bredekamp. Munich: Fink, 2003. 157-76.

much as decimal figures represent numbers, these figures are a numerical language; in as much as we calculate using writing, with the help of numeral configurations, decimal calculation creates an arithmetical problem-solving machine, and is therefore a calculating instrument. Unlike phonetic writing, which is already familiar to us, a Janus-headedness comes into play in the operative writing of calculation: the word 'worm' describes a worm, but it is not a worm; the expression '4 – 4 = 0' not only describes a calculative operation, it *is* a calculative operation. *Description and execution come together in the operative writing of calculation*.

We can also explain this coincidence in a different way. Before the introduction of decimal calculation, numbers were written in Roman numerals. However – since Roman numerals do not create a calculus but are only a numerical language – they needed to be calculated with a concrete reckoning instrument: the abacus. The medium of numerical presentation and the instrument of numerical calculation were realized in different ways. The ingeniousness of the decimal place-value system – making written calculation possible and the abacus redundant – consists of being able to realize the representing of numbers, and operating with numbers, within one and the same system. And this 'double occupancy' of a symbolic praxis – as description and calculation – is a characteristic of a calculus alone. Only the (correctly) calculated expression also carries out what it describes. And this marks the core of its performative dimension, belonging to logic, mathematics, the sciences, to computer programs, or to everyday calculations.

In relation to the calculus, we have so far distinguished two aspects of its performativity. In the course of the *medialization* by the structure-image of the calculized writing, its referent takes on the status of a systematic entity, which is also a theoretical entity. In the course of the *instrumentalization* of the writing-medium as calculating-procedure, presentation and execution coincide.

6. The Example of Zero

Using the example of zero, we want to show how, by the symbolic practice of written calculations in the decimal position system, a new, an epistemological object, is created. The hub of this consideration is a distinction. Normally we assume that the figure '0' 'means' the number zero. Actually however the figure '0' was invented centuries before the number zero was recognized as a mathematical object.[10] The difference

[10] Cf. "The invention of zero preceded its discovery by centuries." Constance Reid.

between figure and number is not simply the difference between a physically visible mark and its 'invisible' referent, but rather the different function which the figure zero adopts in relation to arithmetical signs. Intuitively we can clarify this by two expressions: In the equation '10 – 1 = 9' the sign '0' serves within a numeral-expression to mark an empty place which shows the absence of one of the other figures (1, . . . ,9) in this position; here the '0' functions as a 'gap-sign.' In the expression '0 – 1 = -1,' on the other hand, we treat the figure '0' as an independent number; it functions here as a number-sign. In the first instance '0' symbolizes the absence of other signs, and in the second, the presence of a number.[11]

What we want to show now is that the creation and consolidation of zero as a mathematical entity is not already carried out with the introduction of the gap-sign, it is first ushered in by calculations with the figure '0.' It is therefore a matter of the 'birth of the number zero from the spirit of the cultural technique of the written calculation.'

The Greeks and the Romans did not know a zero. We 'have' numbers only in accordance with a medium, that is, a numerical language which makes the (invisible) numbers appear in a more material form, and then also makes them usable. However, the numerical languages common in Greek and Roman cultural circles – Greek letters and Roman numerals – treat numbers in the sense of a group of countable quantities, and this in such a way as to make calculating with these written numerical signs unsuitable. Such an identification of numbers with 'quantity' is then enforced by the praxis of using a physical calculating instrument while calculating, since the abacus also presents numbers as a collection of singularities. Empty sets, i.e. absent elements, are not countable. Consequently the zero is unknown in Greek and Roman cultures. The reason for this is that their numerical language is not constructed as a calculus; 'calculus' understood here as a written sign-system with the double function of being both the medium of numerical presentation and the instrument of numerical calculation.

The decimal position system was developed in India, where a sign for zero was not only introduced in order to clarify numerical expressions, but where zero was used in calculations, and where rules were also developed for these calculations. The operative impetus to the decimal position system, also in so far as this only served a numerical repre-

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From Zero to Infinity. What Makes Numbers Interesting. San Francisco: Mathematical Association of America, 1992. 1.

11 Reid also distinguishes between zero as a 'number' and as a 'symbol'. Ibid. 6.

a) Roman abacus

10^6	10^5	10^4	10^3	10^2	10	1
M	C	X	M	C	X	I
●		●	●	●	●	
●		●		●	●	
		●		●		
		●		●		
		●		●		
		●				
2	(0)	6	1	5	2	(0)

Indian notation: **2061520**

b) decimal notation

$2061\ 520 = 2 \times 10^6 + 0 \times 10^5 + 6 \times 10^4 + 1 \times 10^3 + 5 \times 10^2 + 2 \times 10^1 + 0 \times 10^0$

according to formation rule

$a_6\, a_5\, a_4\, a_3\, a_2\, a_1\, a_0 = a_6 \times 10^6 + a_5 \times 10^5 + a_4 \times 10^4 + a_3 \times 10^3 + a_2 \times 10^2 + a_1 \times 10^1 + a_0 \times 10^0$

Fig. 1: Roman abacus and decimal notation.

sentation, can be clarified by a simple consideration. Independent of the historical proof of such a connection,[12] the decimal position system can be at least systematically explained as the 'writing down' or transferral of the abacus technique into the medium of numeral-writing where the columns of the abacus remaining empty are marked in the numeral-configuration with '0' (fig. 1).

There is, however, also an etymological connection:[13] 'Sunya' in Indian means 'emptiness' and was called 'Sifr' by the Arabs, which then as 'Cifra'/'cipher (figure)' at first only described the sign '0', but then the whole numeral system. Does the word 'emptiness' play on the missing calculating stones in the corresponding column of an abacus?

12 A historical connection is unlikely. On the "systematic descent" of the Indian decimal system from the abacus, cf. ibid. 2f.; Rotman. *Signifying Nothing*. 10ff.

13 Cf. Joseph Needham. *Science and Civilization in China*. 7 vols. Cambridge: Cambridge University Press, 1959. Vol. 3, 75.

In any case, the trick of the decimal position system consists in being able to carry out the functions of the abacus as pure sign or writing operations, as calculations on paper. And with these calculations it is inevitable that the gap-sign also invades the function of a numerical sign, as soon as for example '4 – 4 = 0' is calculated. The operative certainty inherent in algorithmic calculation prepares us gradually for the interpretation of '0' as a 'positive' number. Nevertheless, the denial of nothingness so deeply rooted in Greek as well as Christian tradition stood in opposition to this. It remains a matter of speculation whether the association, widespread in Indian culture, of emptiness with something that, though it is 'still' nothing, can become something (an understanding of nothing as a potential and proneness) inspired the use of the mathematical zero. In the context of Western thought, however, the resistance to emptiness, the zero, and the vacuum was huge.[14] It is not by chance that the struggle for an experimental proof of the vacuum,[15] and for the mathematical recognition of zero as 'nothing' that is still 'something' go hand in hand within European scientific development. The cultural struggle between 'abacists' and 'algorists' nevertheless spanned centuries; and it was decided less by numerical-theoretical debates, than by the pragmatic demands of soaring trade-capitalism, which knew how to make use of the calculating superiority of the Indian numerals over the Roman numerical sign-system.

Seen in mathematical terms, two 'threshold phenomena' are significant for the '0' to receive the honorary status of a number. Firstly, the *Analytical Geometry* developed by Descartes where zero is placed at the center of the coordinate system.[16] Ever since the Greek discovery of the incommensurability of the side and the diagonal of a square, *multitudo* and *magnitudo*, the countable and measurable, arithmetic and geometry, were treated as mutually non-translatable measures. To the extent that Descartes, with the help of the coordinate system, depicted points on number pairs, he showed the mutual translatability of geometrical figures and arithmetical numbers. Familiar with Indian numeral

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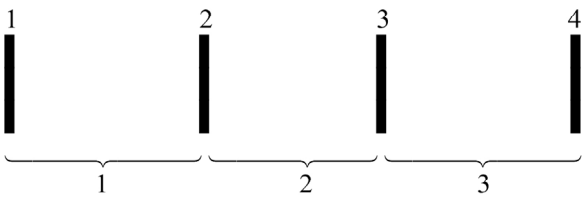
14 Very interesting in relation to this: Robert Kaplan. *The Nothing that Is. A Natural History of Zero*. London: Penguin, 2001. 102ff.

15 Cf. Hartmut Böhme. "Das Unsichtbare. Mediengeschichtliche Annäherungen an ein Problem neuzeitlicher Wissenschaft." *Performativität und Medialität*. Ed. Sybille Krämer. Munich: Fink, 2004. 215-46; Wim Klever, ed. *Die Schwere der Luft in der Diskussion des 17. Jahrhunderts*. Wiesbaden: Harrassowitz, 1997.

16 René Descartes. *Geometrie*. Ed. Ludwig Schlesinger. Darmstadt: Wissenschaftliche Buchgesellschaft, 1981.We neglect the fact that, for Descartes, the coordinates were not right-angled, and that he did not include the negative numbers in his system of coordinates. This was done by his followers.

The zero makes counting and measuring self-referential by marking the starting point of the counting and measuring subject.



Step from the first stick to the last! How many steps? Three!

How can the number of sticks and the number of steps be homogenized?

The first stick is 0.
0 is the starting point of the stepping subject.

Fig. 2: Counting and Measuring.

calculation, Descartes was certain that the coordinate axes, to facilitate this translatability, could not begin with '1.' In this way he avoided a mistake that our calendar still makes: if the birth of Christ is positioned at year 1, then the zero as caesura in the transition from counting the years from 'before Christ' to 'after Christ' is simply missing.[17] How the translatability of the measurable into the countable is made possible by the introduction of zero is made clear by a simple thought experiment (fig. 2).

If we put four sticks next to each other, mark the sticks 1, 2, 3, and 4, and from the first to the last, put each a step apart, then we need exactly three steps to reach the last, the fourth stick. We count four sticks and measure three steps. As soon as we begin the marking of the sticks with 0, so that the last has the figure 3, the quantity of the sticks and the quantity of steps is homogenized.

17 Cf. Dick Teresi. "Zero." *The Atlantic Monthly* 280/1 (1997): 88-94.

With this thought experiment something else becomes clear: the zero represents the point of view, the 'starting point' of the measuring subject. We will come back to this. Now we want to turn to the second mathematical threshold-phenomenon for the transformation of the gap-sign into a number, and that is algebra.

The early history of algebra knew an abundance of particular procedures for solving equations. However there was still no universal method which might, in a general way, express an algorithmic procedure for whole classes of similar equations. Two far-reaching innovations cleared the way for this. François Viète invented symbolic algebra, in which he not only denoted the unknown coefficients of an equation by letters of the alphabet, which was aleady in practice, but also the known. Now it was possible to write down the rules for the conversion of equations with the help of variables in a general way. Like the zero in a numeral configuration, the variables are also 'blanks' and 'placeholders.' However, zero was decisive for the progress of algebra in an even more direct sense. Again the problem to be solved is the universalization of particular algebraic procedures; though now it is no longer the case of the invention of a new 'numerical language' in the form of the alphabetic-writing of symbolic algebra, but rather an algorithmic procedure that consists of solving equations using zero. John Napier succeeded in homogenizing various particular methods by proposing 'equations with nothing' (fig. 3).[18]

Let's stop here. The circle of the mathematical invention, in which the zero plays a meaningful role, does not reduce itself to analytical geometry and algebra. We cannot, however, step out of or get to the bottom of this circle. Instead, we can ask what is shown by these few contexts, sketched by us, where zero plays a role.

7. How Zero as Medium Makes the Heterogeneous Comparable

We are interested in the connection between the appearance of the figure '0' as an indispensable element of a decimal mode of writing numbers, and the emergence of the number zero as a mathematical object. Our emphasis on zero has at least made this much clear: zero *is* a number as soon as the figure '0' *is needed* in mathematical expressions to represent a number. Therefore with the number zero, an epistemological object appears whose existence as a referent of the figure '0' is due

[18] Kaplan. *The Nothing that Is*. 140ff.

a) al-Hwarizmis equation, since 825 in countless textbooks

$x^2 - 39 + 8x = -2x$
$x^2 + 8x + 2x = 39$
$x^2 + 10x = 39$

<u>There were three particular methods</u>
$x^2 + px = q$
$x^2 + q = px$
$px + q = x^2$

b) Equation with zero: John Napier (1550-1617)

<u>Now universal method</u>
$x^2 + 10x - 39 = 0$
$x^2 + 10x - 39 = (x - 3)(x + 13)$

according to: if ab = 0; then a, b or both = 0

$x - 3 = 0$ or $x + 13 = 0$
$x = 3$
$x = -13$

Fig. 3: Algebra.

to the cultural technique of written calculation. Thinking further along these lines, we could, in good post-structuralist manner, discuss how, through movements of the signifier (the figure '0'), the signified (the number zero) is first of all crystallized; and how the materiality of a symbol, which can be used palpably, has a decisive effect on its semanticity. In short, how the use of a sign first of all creates its meaning (also in the referential sense of a relation to an object) can be demonstrated here almost prototypically. With such arguments we remain bound (as, incidentally, Brian Rotman is in his stimulating thoughts on zero[19]) to a semiotic perspective where anything interesting to be said about zero takes place in the exchange-relationship between sign and referent, and consequently obeys the semiotic regime, and the logic of the sign.

However, it is precisely the anchoring of our considerations concerning zero within cultural techniques that has shown that what the figure '0' does, is not only – though, naturally, also – the creation of a

19 Rotman. *Signifying Nothing*.

new object, but also the provision, perhaps even 'creation,' of a flexibility for the calculating and mathematical act. If we examine closely what this flexibility consists of, it becomes clear that the operative spaces opened up by zero make it possible to connect formally separate or various entities and to homogenize them, and consequently to set free new kinds of calculating and mathematical practices. If this view can be confirmed, then the figure '0' appears less as a sign-carrier, with the peculiarity of preceding by centuries its 'belated' referent, but rather as a *medium*; 'medium' understood here as a 'middle' and 'mediator' which makes heterogeneous worlds relatable, and mutually translatable. The supposition is therefore that the revolutionary potential of zero consists in its function of *becoming productive as a medium*. And that this media-technical potential distinguishes the zero from the other numerical signs of the decimal system – the zero fulfils a function here which cannot just as well be realized by the figures '1,' '2,' '3,' . . . – although the zero can only develop its efficacy as an element of decimal calculation. To verify this thesis would require a thorough investigation which cannot be carried out here. We can only suggest in a brief outline why we think a media-view provides an interesting potential elucidation.

The media-concept, which we consequently 'bring into the discourse,' can only be supposed, not deduced. Media mediate between heterogeneous fields or systems or worlds by enabling a transferral or exchange between two divergent sides, and as a result open up new space for cultural practices. Media can do this by embodying, by hybridization, attributes from both sides of that which is to be mediated. In this way they provide a connection between diverse things without having to surpass and annul this diversity.

And now back to zero. The role of the figure '0' as gap-sign in the decimal position system can be reconstructed in such a way – we have already demonstrated the beginnings of this – that the 'empty place' (*leere Stelle*) in the abacus is transferred into a 'blank' (*Leerstelle*) within a numeral configuration. For this transferral, two things are significant. While on the abacus, in a particular column, there are *no* stones, in the numeral configuration the missing and absence of something is marked with a sensibly perceivable sign. Nothingness is quantified;[20] the emptiness is aistheticized. Exactly this possibility of making absence not only sensibly present, but also quantifiable and materially

[20] On the role of the quantification as the cardinal attribute of modern European thought, cf. Alfred W. Crosby. *The Measure of Reality. Quantification and Western Society 1250-1600*. Cambridge: Cambridge University Press, 1997.

usable, that shows that not merely does a blank on the abacus wander into a writing configuration, but also that a technical function ('calculate with calculating stones') is transferred to a symbol system ('operate with signs'), with the performative effect that now the presentation of numbers is also a production of numerical relations; since calculation is nothing but this. The systematic connection between abacus technique and decimal calculation shows that the mediating function of zero consists in creating a hybrid which simultaneously makes technical and symbolic functions possible, and, with this pairing of technology and language or writing, in creating a medium that goes much further than the preceding calculative systems, as well as numerical systems.

Or, let's consider the coordinate system that becomes a medium for connecting geometry and arithmetic, the measurable and the countable. The zero is situated at the point of the intersection of the axes. This position is normally characterized by the zero being happily designated as the origin of the number lines that cross in it. Without wanting to discredit this origin-idea, we might characterise the place of zero less as a beginning, but rather as a middle, and a middle-point. By virtue of this placement the zero can set in motion an exchange between the space of the negative and positive numbers, between the present and absent, it makes a transition from the axis of positive value to the axis of negative value possible. Zero provides the connection between the y-axis and x-axis, and consequently can translate geometric points into combinations of the values of *both* axes. In any case, with this 'reifying' mode of what the zero does, we must constantly keep in mind that it is human or mechanical computers that put this, with the help of zero as the medium, to work. What the hybrid character of zero means here becomes clear with the example of the coordinate system. Zero is as much the beginning of the positive numbers on the x-axis as the negative numbers on the x-axis; it is the beginning of the positive and the negative y-values. What this comes to is not simply that it is a beginning, but rather that zero embodies the beginnings of two axes *at the same time* and *in one*. This 'in one' cannot be valued highly enough. By this kind of hybridization of two sides, which in a certain way neutralize each other, a kind of indifferentiality of the zero is achieved, the kind of neutrality, incidentally, which is concerned with maintaining the operational logic of each medium.[21] In this way, zero clarifies what it means when, to make distinctions, we continuously require a medium. The zero marks a line

21 Zero is therefore not simply nothing, but according to 4 – 4 = 0, what results when two inverse procedures are used.

of division that enables the distinction between two heterogeneous areas, to the extent that, in the medium, the zero can bring about the different and the common between what are distinguished. By 'sitting' in the middle of the number-series which divide the negative and the positive numbers, the zero also shows that, with the serious difference between having and not having, between presence and absence, between negative and positive in the homogenous countability provided by the zero, those things which diverge from each other are comparable and can be related, a countability and comparability, which now, for example, allows the question: what is the result of 'seven subtracted from five'? It is the appearance of zero that makes this transition possible at all.

8. The Central-Perspectival Vanishing Point as Visual Zero

Let's turn now to the background of the above-mentioned media-theoretical perspective on the early modern period which – and actually significantly earlier than the zero became meaningful in the mathematicizing sciences – made the zero-point the increment of its central perspective pictures. This is the case of the vanishing point, that point within a picture constructed using central perspective, which is infinitely distant from the observer.[22] In a picture not governed by perspective, the pictorial space is an associative space of objects whose mutual relationship is best defined by the meaning which these respectively embody: the important things are depicted as big, and those second in rank are depicted as smaller. By means of the vanishing point, the pictorial space is transformed into a system organized by coordinates in which the proportion of each object is calculable. The nearer the objects in the picture are to the vanishing point, the smaller they are in scale, and the further they are away, in the 'fictive reality' of the picture, from the viewer. It is no longer the meaning, but the relation between the 'reality' presented in the picture, and the viewing-process of the viewer, which produces the matrix of the coordinate system. What the vanishing point means can partly be understood in the way that it shifts the viewer into the picture as organizing center. These associations are familiar enough. Therefore we want to concentrate here on the question of what is actually gained as soon as the vanishing point is understood as a 'visual zero.'[23]

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22 Cf. Rotman. *Signifying Nothing*. 14ff.; Crosby. *The Measure of Reality*. 165ff.; Charles Seife. *Zero. The Biography of a Dangerous Idea*. New York: Viking, 2000. 95ff.

23 Rotman. *Signifying Nothing*. 19.

To begin with, there are certain basic affinities between the point and the zero: points are zero-dimensional objects. What that means, we can clarify operatively by imagining successively robbing three-dimensional objects of their dimensions.[24] First a three-dimensional object is pressed flat so that it mutates into a two-dimensional plane, consisting of length and breadth. Then this flat structure is put on one of its sides, and pressed flat again, so that only a one-dimensional line exists. If this is pressed flat lengthwise, a zero-dimensional point results, which is without extension, that is, without height, breadth, or length.

This is a characteristic of all points. What, however, characterizes the special relationship between the vanishing point and zero? Here too, an operative consideration can be introduced; though in this case not a thought-experiment but an actual experiment that the architect and sculptor Fillipo Brunelleschi staged in 1425. With this experiment, Brunelleschi wanted to document the perfect illusion of the central perspective picture, i.e. to show that our natural perception of world-objects, and the appearance of pictorial objects painted in central perspective, coincide. Placed in a row are: the observer, a picture of the baptistry in Florence turned towards the viewer, a mirror, and the baptistry itself. Brunelleschi drilled a hole in the picture so that an eye of the observer looking through this hole sees the reflection of the pictorial baptistry in front of the real baptistry, and in this way notes that he cannot distinguish between picture and reality. On the mirror surface, the usually invisible vanishing point of the picture is represented or visualized here by the eye of the observer. In 1435/36, Leon Battista Alberti theorized this experiment. He showed that a correct perspectival image could be constructed with a single vanishing point, which is placed in relation to a visual pyramid, whose tip lies in the eye of the viewer, and whose base in the object. The central-perspectival picture could then be defined as an even section through the visual pyramid.[25]

We analyzed the numerical zero in its quality of being a medium that provides a comparability between heterogeneous mathematical domains, and in this way enables 'transitions' between them. In analogy to this, we can interpret the visual zero of the vanishing point as a mediator between the real space and the pictorial space, between the space of objects and the projection space, between the observing subject and the observed object, between the I and the world. The vanishing point structures the picture like a coordinate system, whose zero-point it is. It

24 Cf. Seife. *Zero*. 96.

25 Leon Battista Alberti. *On Painting*. New York: Penguin, 1991.

is not by chance that part of the elementary exercises of perspective construction are tile patterns, reminiscent of coordinates, which in pictorial space undergo a mathematically exact transformation, and precisely for this reason appear as an illusionist development of the natural floor on which the viewer stands. In relation to what is depicted in the picture, mediated by the vanishing point, bodies and the intervals between them can now be handled in a unified way; body and non-body are, despite their qualitative difference, calculable with measure and number. Their size in the picture continuously decreases as it goes backwards, meaning in the direction of the vanishing point, whether it is a case of the objects or their interstices.

Just as various axes come together by crossing each other at the numerical zero at the center of the analytical coordinate system, the projection lines meet at the visual zero. But not only that; above all, the organizing principle of all visible objects – which, like the vanishing point itself, is invisible – coincides with the position of the eye of the observer – and precisely this brings Brunelleschi's mirror to mind. Brian Rotman imagines in the vanishing point the "visual equivalent of a demonstrative pronoun."[26] This is how a subject standing in a totally defined place, now and here, sees the scene. Embodied in the vanishing point, the subject is in the picture. Consequently, Jean Pélerin Viator has perceptively described the fixed point of the eye as 'subject.'[27]

9. The Zero as Embodiment of the Modern Subject?

Dirk Baecker establishes that at the zero, not only counting is reflected, but also the counters themselves, so that the history of the zero can also be understood as the history of the "hesitant discovery and the engaged denial of this observer."[28] In fact, not only in the case of the observer, but also for the subject is "the zero written on the body."[29] The numerical zero symbolizes the standpoint of the measuring and counting subject just as the visual zero marks the standpoint of the observing subject. In this way, an artistic construction principle anticipates what in

26 Rotman. *Signifying Nothing*. 19.

27 Emanuel Alloa. *Diaphanes*. *Vom Begriff zum Phänomen der ikonischen Differenz*. Master's thesis at the Institut für Philosophie of the FU Berlin, 2003. 19, who cites here Hubert Damisch. *L'origine de la perspective*. Paris: Flammarion, 1987. 14.

28 Dirk Baecker. "Der Nullpunkt." Preface to the German edition of Brian Rotman. *Signifying Nothing*: *Die Null und das Nichts. Eine Semiotik des Nullpunkts*. Berlin: Kadmos, 2000. 9.

29 Ibid. 10.

modern philosophy and science since Kant has been apostrophized as the 'Copernican revolution' – that we (only) experience the world as it shows itself from the subjective standpoint, meaning the perspective of the single viewer. The world of objects appears as a projection of the order of the subject.

But is the role of zero actually appropriate for such a constitutive-theoretical, or at least constructivist-oriented view, of humanity's place as subject of the world? We have seen that we can consider zero from a semiotic and medial point of view. Let's try once again to focus the difference of both modes of viewing. Semiotically, the figure '0' appears as a gap-sign in the position system, whose calculating function finally leads to the crystallizing of a new mathematical object: the number zero. The nothingness of emptiness becomes with this definition a something, which is also operable on paper, and, in the same way as the other numerical signs, thus prepares the ground from which the number zero can eventually grow. However, what is here considered in a prototypical way, using the figure '0', is also the case for the other number-definitions. Not only the absence of a specific number, but rather *all* numbers are entities which are *per se* invisible, and which can only be included in the regime of visibility by means of numerical signs. That the visualization of something invisible in its cultural-technical use leads to the visualized arising as a mathematically recognized object, is a 'fate' affecting all numbers in the early modern period, which are no longer, as in Euclid's time, treated as countable quantities of units, but rather as something that can be introduced as a referent of a calculated arithmetical expression. The signs create the objects that they name. From this semiotic perspective a performative constitution of mathematical objects by the use of a formal language can be clearly shown; and the debate on linguistic performativity is enriched with an important and previously neglected aspect.

With this creation of an epistemological object using the cultural-technical practices of sign-use, the epistemological role of the subject as creator of the world, which they can experience and investigate, seems to be confirmed once more. But doesn't this connection show itself in another light when we adopt the media-perspective?

From this media-perspective, the new, which is bound with the figure '0', consists less in the creation of a new object. For media mediate between diverging domains by making them – precisely in their differentiation and despite their differences – comparable, and therefore opening up space for action whose productivity is rooted in interdependencies and new types of transaction between these domains. Before this ho-

rizon, the figure '0' emerged as that which mediates *between* calculating technology and numerical language, consequently bringing about written calculation, or which mediates *between* point and number, and so creating analytical geometry with its potential to solve geometrical problems algebraically.

Brian Rotman has analyzed the zero semiotically and interpreted it as a meta-sign, a sign about a sign, which creates from the "emergence of a meta-subject,"[30] an 'original subject.' Here, two different genitives of 'subject' are quoted, whose operative power consists of creating its sign-systems, and consequently also of being responsible for the emergence of the object which is signified by them.[31] The enlightenment, which this semiotic perspective conceals, lies in a critique of the opinion that the ontology of objects precedes the construction of signs for these objects. This critique is fruitful, though it also draws on the traditional pattern of the subject as creator of the world. But what happens if in a media-theoretical orientation we understand zero as a mediator between diverging worlds, rather than as a mechanism for creating new worlds? What consequences would this have for a conception of the subject, since for everybody who is engaged with the zero, it is at least clear that zero is associable with the standpoint of the counting, measuring, observing subject? Could it be then that a new light would fall on that which defines human activities? Activities, whose ingenuity would not simply consist of making new things, but of offering new connections, from whose power new types of communicative and cognitive operations could emerge? With these questions, this text ends, which has nothing more in view than to show 'family resemblances' between science and art, using the example of zero.

30 Rotman. *Signifying Nothing*. 27.
31 Ibid. 93.

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