Advanced statistical mechanics II

(SS 16, FU Berlin)

Problem sheet 2

Due date: May 10, 2016

Problems

4. **Entropy change (1+1+1+1+1)**
We want to calculate the change in entropy ΔS as a function of the initial and final volume and temperature of a substance while it performs work according to $\delta A = p\,dV$.

   a) Starting from Gibbs' fundamental equation find two functions f, g depending on V, p, and T, such that
$$\mathrm{dS} = \mathrm{f}\,\mathrm{dV} + \mathrm{g}\,\mathrm{dT}. \tag{1}$$
Hint: Consider the internal energy U as a function of V and T.

   Now assume that the substance is an ideal gas of N particles, i.e., $p\,V = N\,k\,T$ and $U = \frac{3}{2}N\,k\,T$. Calculate the change in entropy

   b) ΔS_1, during an isothermal ($T = \mathrm{const.}$) expansion from V_- to V_+,

   c) ΔS_2, during isochoric ($V = \mathrm{const.}$) heating from T_- to T_+,

   d) and ΔS_3, during an isobaric ($p = \mathrm{const.}$) expansion from V_- to V_+ ,

   solely as a function of N, k, and the given parameters respectively.

   e) Calculate the change of entropy during the isochoric heating of $1\mathrm{mol}$ of an ideal gas from 0°C to 100°C.

5. **Maxwell relations (1+2+1)**

   a) Consider the internal energy U as a functions of its natural variables S and V and derive $(\partial U/\partial S)_V$ from Gibbs' fundamental equation.

   b) The free energy F is defined as the negative Legendre transform of U w.r.t. S,
$$F := U - TS. \tag{2}$$
Write its differential dF in terms of dV and dT.

   c) Conclude that
$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V. \tag{3}$$

6. **Entropy of an ideal gas (1+2+1+1)** Consider an ideal gas where
$$pV = NkT \qquad \text{and} \qquad U = \frac{f}{2}NkT, \tag{4}$$
where k is a constant defining the temperature scale.

   a) Conclude from Gibbs' fundamental equation or from the solution to Problem 4a that
$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V. \tag{5}$$
and calculate the heat capacity
$$C_V := \left(\frac{\partial U}{\partial T}\right)_V. \tag{6}$$

b) Show that for some curve c from (T_0, V_0) to (T, V)

$$\Delta S := \int_c \mathrm{dS} = \mathrm{Nk}\ln\left(\frac{\mathrm{T}^{\mathrm{f}/2}\mathrm{V}}{\mathrm{T}_0^{\mathrm{f}/2}\mathrm{V}_0}\right). \tag{7}$$

Hint: The result from Problem 5 can be used.

c) Argue that there must be a function depending only on N such that

$$S(T, V, N) = Nk\ln\left(\frac{T^{f/2}V}{f(N)}\right) \tag{8}$$

d) From the fact that V and N are extensive variables of S, i.e. $S(T, \lambda V, \lambda N) = \lambda S(T, V, N)$ for any $\lambda > 0$, find that

$$f(N) = \Phi N \tag{9}$$

for some constant Φ (that is independent of all the variables). What is the final result for the entropy of an ideal gas of N particles at temperature T in a volume V?

7. **Clausius' statement (3)** says that "heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time." Prove this statement by showing that such a process could be used to build a perpetual motion of the second kind.
Hint: It can be helpful to use a diagram. Nevertheless give all equations necessary for a complete proof.

Informations

- The problem sheets will be distributed always on Tuesdays.
- Each problem sheet will allow for earning 16-20 points. The cool questions will be marked as bonus and will allow you to get even more points!
- The solutions can be handed in by groups of two students. You will have to present your solutions in the tutorials so both of you have to understand the solutions.
- You are permitted to work together with more students, but then indicate all their names on your completed homework.
- The solution is to be handed in right at the end of the lecture on the Tuesday one week after we have handed out the exercise. If solutions are handed in too late you will get zero points for your solution.
- Please indicate the tutorial in which you participate on your solution.
- It is necessary to have achieved at least 50 percent combined on all exercises to pass the tutorials.
- Tutorial page: `http://userpage.fu-berlin.de/~marekgluza/ASM2_16/` (short link: `bit.ly/1XTWwvn`)