1 Embedding of First-Order Nominal Logic into Higher-Order Logic

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Nominal logic, also referred to as hybrid logic, is a general term for extensions of ordinary modal logics that introduce a new sort of atomic formulae, the so-called *nominals*. These nominals are only true at one possible world and false at every other world. The *shifter*, denoted @, can be used to evaluate the truth of a formula ϕ at a given world corresponding to nominal i as in $@_i\phi$. The simplest of those nominal extensions including the above ingredients is often denoted $\mathcal{H}$ [4]. Early nominal logics originated from Arthur Prior's research on *tense logics* [7] and were, since then, heavily researched, most notably by Robert Bull, Robert Blackburn [4] and Valentin Goranko [9].

Classical higher-order logic (HOL) (also known as Church's Simple Theory of Types [5]) is an expressive formal system that allows quantification over arbitrary sets and functions. Its semantics is meanwhile well-understood [2] and several sophisticated automated theorem provers for HOL (with respect to Henkin semantics) exist (e.g. *LEO-II*, *Satallax*, *Isabelle*).

We employ an embedding for quantified (first-order) ordinary modal logic into HOL (see [1]) and appropriately augment it for the sound and complete embedding of first-order nominal logic (FONL). The embedding approach suggests that FONL can be regarded a fragment of classical higher-order logic and thus allows the out-of-the-box automation of the here discussed nominal logic using common HOL automated theorem provers.

As a proof of concept, we implemented the above embedding for *first-order nominal tense logic*, a quantified version of the nominal tense logic used by Blackburn [4], as a theory in Isabelle/HOL and a TPTP problem file in THF format [8]. The embedding implementation was successfully applied to automatically prove certain correspondence theorems between frame conditions and axiom schemes, including those of regular modal logic and those which can only be expressed within the extended expressivity of nominal logic [3].

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Further experiments could be expanded to more complex applications, e.g. for formalizations from the field of linguistics and philosophy.

The self-imposed restriction to first-order quantification within the embedding is somewhat artificial: Second-order or even higher-order quantification could easily be reduced to the quantification of the HOL meta-logic. But since the semantics of higher-order notions of nominal logics is not immediately clear, the formal treatment of higher-order quantification in the nominal logic context is further work [6].

Already existing automated theorem provers, such as *hylotab*, *htab* and *spartacus* are restricted to propositional nominal logic and subjacent modal logic K. In contrast, our embedding allows the flexible adjustment of the underlying modal logic (by imposing frame conditions at meta-logic level) and the addition of further modal operators.

References

1. C. Benzmüller, L. Paulson, "Quantified Multimodal Logics in Simple Type Theory", *Logica Universalis*, 7(1):7–20, 2013.

2. C. Benzmüller, D. Miller, "Automation of Higher-Order Logic", *Handbook of the History of Logic*, vol. **9**, Elsevier, 2014.

3. P. Blackburn, "Representation, reasoning, and relational structures: a hybrid logic manifesto", *Logic Journal of IGPL*, 8(3):339–365, 2000.

4. P. Blackburn et al., "Handbook of Modal Logic", *Studies in Logic and Practical Reasoning.* Elsevier Science, 2006.

5. A. Church, "A Formulation of the Simple Theory of Types", *J. Symb. Log.*, 5(2):56–68, 1940.

6. M. Wisniewski, A. Steen, "Embedding of Quantified Higher-Order Nominal Modal Logic into Classical Higher-Order Logic", *Proc. ARQNL*,2014.

7. A.N. Prior, "Papers on Time and Tense", Oxford books, 1968.

8. C. Benzmüller, F. Rabe, G. Sutcliffe, "THF0 – the core of the TPTP language for classical higher-order logic.", *Proc. IJCAR*, 2009.

9. V. Goranko, "Temporal Logic with Reference Pointers", *Proc. ICTL, LNAI 827*, Springer, 1994.