MATHEMATICS IN THE HISTORY OF PSYCHOLOGY

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If we do not come up to more, I would be glad to transmit the following two things: firstly there are not only the mathematical sciences which meet with problems on the occasion of their attempt to relate their history with the kuhnian pattern of interpretation, because the same thing is happening and even in a more severe way with the social sciences and particularly with psychology. Secondly it is just mathematics which due to their very special role in the framework of psychology are contributing heavily to this distortion with regard to the kuhnian system of reference.

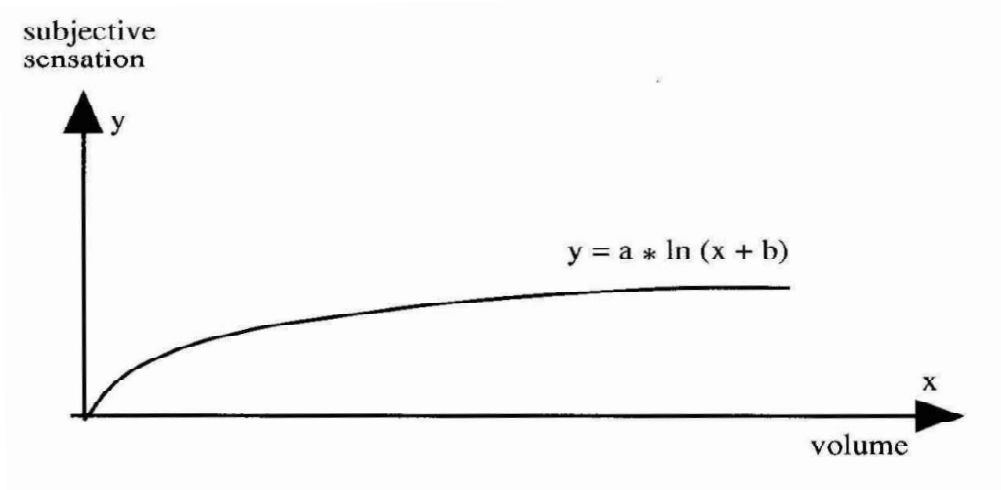
Speaking about the history of psychology always turns out to be a delicate thing. For there does neither exist conformity between the representatives of this matter as to the beginning of this history nor as to the subjects that it should include.

It happens to be even more controversial to speak about the part which mathematics are playing within this history. For the most influential protagonists of contemporary psychology take mathematization as an intrinsic tendency of all the sciences and considering mathematics as the last resort of truth, there does not remain any margin for a critical investigation relative to this role and its changes. To such a degree that for these people talking about mathematics in psychology is the same thing as talking about progress and so that's that.

Nevertheless we will now risk a glance at the history of psychology and the part that mathematics are playing there.

The first contributions to modern psychology go back to the 19th century and just to scientists working in the field of classical natural sciences and well acquainted with mathematics. Among this kind of men is ranking the physicist G.Th. Fechner (1801-1987) and the physiologist E.H. Weber (1795-1878), both founders of the so called psychophysics. Its programme consisted in establishing regular relationships between physical scales (e.g. the weight of an object) and psychical scales (e.g. the just noticeable difference between weights which is necessary to distinguish them).

It is only natural that in the context of this kind of study the question of the utility of mathematical functions suited to model the curves which the experiments yielded pretty soon rose. So that already at the beginning of modern psychology the people made use of mathematics in a really amazing manner. Let us take the famous Law of Fechner which held a logarithmic relationship between physical variables (the independent variable, e.g. the volume of an acoustic stimulus) and the intensity of the subjective sensation (the dependent variable, e.g. the acoustic reception of the same stimulus). By the way, it’s simple to derive this relationship on the basis of the already mentioned just noticeable differences by means of the differential or difference calculus, respectively. The following graph shall serve to illustrate this kind of psychophysical laws.



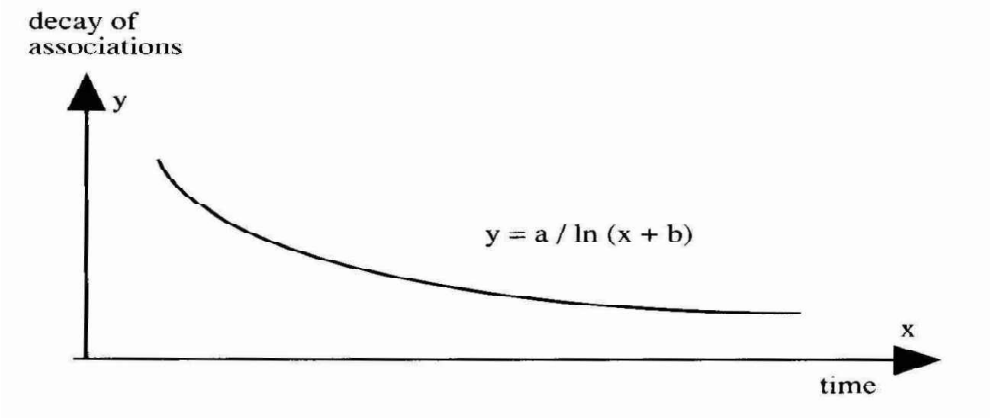
For a rather long time psychology followed this course accumulating this type of regular relationships which characterize psychical mechanisms, going more and more beyond the sensorial functions which formed the specific scope of psychophysics. Another prominent representative who marks this more extensive programme in which the original psychophysics ended up was W. Wundt (1832-1920). His laboratory, founded in 1859, is taken in general to be the origin of modern psychology in its proper sense. Wundt, philosopher, carried through a systematic generalization of the project of a forthcoming scientific psychology. According to him it should devote itself to the explication of psychical phenomena basing it on a set of primary elements which by the way of combining themselves end up in more complex arrangements held together by some principles of association. Due to this background the historians of psychology used to name this project the *psychology of elements* or *psychology of association.*

What matters in the present context is that questions of this character almost automatically give rise to mathematical concepts. For example one problem to deal with was *what is the function like which could describe the relationship between the number of elements (say senseless syllables) and the number of presentations of their corresponding arrangements necessary to obtain stable associations?* To get a more vivid idea we can imagine the following pairs of senseless syllables:

yer-fis dak-mun per-xol hen-sof get-tok

which will be presented as many times as we need to come to a correct reproduction.

Or, once we have reached such stable associations with regard to a given arrangement: *what is the curve like which could describe their decay taking place in the course of time?* Later on this kind of problem led to the investigations of H. Ebbinghaus (1850-1909), a disciple of Wundt, who was intimately engaged in the study of the *mechanisms* of memory. The following graph shows one of these laws of memory.



As a matter of fact this sort of experiment already requires more than one person and something like statistical techniques. For given the poorly standardized conditions of the experimental setting and the imponderabilities of the subjective dynamics on the part of the investigated person as to the results we always will remain with a considerable fluctuation. But the above mentioned scientists maintained that fundamentally and under perfect conditions one can do with a single subject to obtain all the data proper for generalizations relative to the human psyche. Preferably it should be the scientist himself as the best trained person in inspecting his own psychical apparatus. Clinging to this *pure-case-concept* one can only attribute a marginal relevance to the individual differences, while the real thing that counts is the general structure of the human psychical organization. The object of psychology according to this comprehension is to elucidate the essential characteristics of this general structure by means of theoretical models in which mathematics at the most can play an auxiliary part.

This view of psychology persisted for a rather long time and brought forth other schools like the school of Würzburg (to whose more famous representatives K. Bühler and 0. Külpe belonged). At the same time, this means at the beginning of the century, this concept of psychology spread from Europe to North America where, under the protagonism of E.B. Titchener (1867-1927), it adopted the name *structural psychology.*

To establish a first reference to the kuhnian model of scientific development perhaps we could speak of a pre-paradigmatic phase of psychology whose typical feature was an assembly of individual researchers inspired by their respective idiosyncrasies and at most united by invoking a concept of scholarship derived from the classical sciences. Far from any pretension to enter into questions of obligatory norms to be observed in the theoretical and practical work of psychology they managed to coexist peacefully in their respective *ivory towers.*

But shortly afterwards there was a radical break in the evolution of psychology so far described, a break which was initiated by the functional manifesto of North American psychologist J.R. Angell (l869-1949) [ANGELL, 1907] and later intensified by another manifesto by J.B. Watson (1878-1958) [WATSON, 1913] in favour of a total abolition of the psychology which existed until then and its substitution for behaviourism.

More concretely: what impact did this break have on psychology in general and the function of mathematics within it? It is not feasible to analyze this problem without going back to some factors outside psychology that formed the historical background of this evolution. In the scientific and philosophical field we must mention the trancendental influence that came from Charles Darwin's ideas and the biological and evolutionist thinking inspired by them. With relation to human psychology it would mean that psychism is nothing more than another result of an ubiquitous adaptation. Going beyond the human species as

such it would suggest that each individual, far from being a realization of the human being, characterized by a general structure is a different product of its own adaptive processes and more or less accurate according to each case.

However, as far as psychology is concerned Darwinism has another effect: there is not any longer a coherent object left for psychological research and the theories that arise from it, but this is converted into a population of exemplary individuals which comprise a variety of characteristics and levels of adaptation. In this population man and his psychism as the objective of the psychology that existed until then is diminished to the size of a dot, which not only implies the dissolving of the scientific objective but also the corresponding theoretical pretensions. From now on the principal objective of psychology consists in researching the adapting value of those characteristics and in identifying the most adapted of fellow human beings to distinguish them from the less adapted ones. So, Angell's term functionalism refers to this programme of questioning the usefulness of psychic mechanisms as adaptive functions instead of going into its structures and lawful characteristics. And if this weren't enough later on this last type of questioning is denounced as being a useless luxury and moreover a sort of very little scientific obscurantism. What happened was that this programme was not even begun because functionalism came into the shadow of behaviourism, its more radical child. At the same time behaviourism proclaimed the definite renouncing of all theoretical pretensions in favour of total empirism being content with finding data about the observable responses provoked by an observable stimulus considering the psychic processes either non-existent or like a black box which is out of scientific reach.

To establish another reference to the kuhnian concepts of scientific evolution: how to expect the shaping of a paradigmatic basis for psychology faced with a complete renouncement of a theoretical body and therefore, a sufficiently exhaustive delimitation of the object under research? However, there was a *hegemony* indeed that in the United States for many decades succeeded in excluding any other way of focusing the *official* argument of psychology. So, the sociological factors subjected in the kuhnian analysis to maintain a hegemony determined the state of things but without the counterbalance of having to offer solutions to both theoretical and practical problems put forward by the object of research. In that sense we can talk about a hegemony without any rational basis. This hegemony continues to the fore in many Latin American countries.

To get back to the second part of our question: what does all this mean to the use of mathematics in psychology? In fact, it was precisely a kind of logical mathematical background which brought about this hegemony, namely a terminology inspired by the logical empirism of the Vienna Circle, the physicalism and the cult around a total abstraction which finds its ideal in mathematics and formal logic. So paradoxically it was ideologically disfigured mathematics which gave rise to this irrational and intangible basis of behaviourist hegemony.

From time to time attempts were made to take these logic-mathematical pretensions seriously. Here it's worth mentioning E.C. Tolman (1886-1959) and his topological model to represent the *mental maps* of behaviour and more particularly we mention C.L. Hull (1884-1952). The latter spent his life desperately designing a mathematical apparatus capable of representing the afore mentioned black box based on the so-called intervening variables in order to reproduce his experimental data obtained from his rats [HULL, 1952]. However, this project as the mathematical apparatus was inflated more and more, ended in a total disaster. This marks the beginning of the decomposing of the behaviourist hegemony which finalized breaking up into several intricate sects mutually at war.

As we once again establish a reference to the kuhnian concepts it was not in the first place due to an internal crisis that this behaviourism broke down as the impossibility of solving research problems out of the reach of a paradigm which until now had been successful, but due to outside intervention. It was the famous linguist Noam Chomsky (born 1928) who, in 1959 took the whole argumentative basis of behaviourism to pieces in a review of one of Skinner's texts, contrasting his position with some elementary facts of linguistics. This caused a general upset in North American psychology which had repercussions in the whole world and was followed by a precarious phase in which behaviourists found themselves devoid of their ideological clothing. Some of the supporters joined ranks with the Chomsky project and the others in search of a substitution to Skinner's abstractionism and his compromise with a logic-mathematical vision of the world fell in with the cybernetic situation of these years under the lemma of cognitionist change.

It was another attempt to forcefully adopt a language outside psychology as it happened formerly to the language of the classical sciences to try to find some watchword of scholarly method. This implied a fundamental inversion of means and ends: the cybernetic machinery does not serve as a useful model to represent some characteristics of the cognitive processes by means of simplification and abstraction and bearing in mind their character of a tool but quite the reverse: the mind *is*

nothing more than a computer and methodical abstraction to arrive at this analogy it is converted into an ontological quality. And thus a fetishization of models is produced. The cybernetic categories substitute a genuine psychological language or *epistem* in such a way that the mind is seen as a place where abstract algorithms of the cybernetics are manifested. In effect psychology continues to ignore concrete manifestations of the human psyche blowing up what were the interior of the empty points which behaviourism had made the human individual take on in order to fill later on up the theoretical vacuum with cybernetic constructions. Once again there are hardly any *practical* benefits for this operation which would compensate for *theoretical* deformations unless we take as such the ideological adaptation of our society to the machinery that dominates it. Here I make reference to the philosopher Günther Anders who critically focuses the relationship man-technology and man's inferiority to it.

Although the project pointed in some way to a recuperation of the behaviourist scheme (relate the input of the human system to its output) [LEAHEY, 1991] the abandoning of the critical postulate of man-black box opened the way to conceptual eclecticism which consisted in mini-theories or individual hypotheses on a phenomenon without any coherence and even less a totalizing focus. In consequence a further detonation took place within the greater part of the researchers as a result of the absence of a structural principle to define the object of their study. This gave rise to what we call the era of *statistics* and the *analysis of the conditions* coinciding with an almost total dominance of the methods toward the object. It is not a case of studying empiric data with an explanatory scheme but of producing data with which to feed a statistical model. So the predominance of methods is really nothing other than the predominance of mathematics.

The central point of dominating mathematics is mathematical statistics which elaborated one model after the next in which each one corresponded to an experimental design. As there is a never-ending variety of possible experimental designs with relation to independent and dependent variables we have a never-ending story. Mathematically most of these statistical models are based on the lineal premise that is to say the lineal combination of effects of the different variables something that is totally distant from the complexity of psychic processes.

But this is of no importance, we are trying to fill a theoretical and conceptual vacuum and as the lineal models open up an impressive mathematical world, they fulfil the function of a theoretical and conceptual substitute.

In addition we must draw our attention to the enormous abstraction which the conversion of human beings in examples of a statistical population and finally in the elements of a statistical survey involves, something which foments this fundamental inversion in the relationship between the means and ends. The statistical models are not the means used to represent the end of a real object, but the statistical models are converted into ends and an experiment is used to demonstrate the sophisticated character of the model in question. More concretely it means that a colleague first becomes acquainted with a new mode of analysis of variance and then looks for an empiric problem to which he can apply his model.

It is not by mistake that we now speak in the present tense since this era persists and with it this theoretical vacuum has meanwhile transformed itself into something like an epistemological virtue based on a vague interpretation of critical rationalism. In accordance with this interpretation it is sufficient to accumulate a large amount of results through experiments as contradictory or isolated as they may seem, and finally a theory will appear in sight. That's why methodology, without its conceptual legitimization which behaviourism gave it relating it to its physicalist vision of behaviour conditioned by the environment, has become dominant without any precedent in modern day psychology. Although from time to time theoretical flashes occur they generally consist of disfigured adaptations of mathematical concepts to psychological questions.

It is worth remembering here the deplorable history of theories on the structure of intelligence based on factorial analyses (a technique related to reduction of square root form more common among mathematicians under the term of transformation of the principle axis in a vectorial space). To show one example:

According to a forerunner of this current theory, a certain Adolf Otto Jäger, seven factors of intelligence of universal character are to be found in the entire world and in all cultures. He also maintains the belief that all individuals can be characterized by more or less invariable values with respect to these seven factors [JÄGER, 1984]. It is clear that this belief is beginning to become displaced if we start with the concrete context in which intelligence is used as a rule, and we discover that the structure of such processes very much depends on the type of problems on the one hand and the strategies and tools available on the other. To mention a more concrete example: nowadays, because of the presence of computers in our lives and that of children, algorithmic techniques are becoming more and more common as compared with 30 years ago, when they were almost unknown. It is not difficult to understand these implications from a rational point of view. But logic has inverted in this type of psychology. The starting point is the abstract idea of the seven factors

and in order to defend it all available manipulations can be used and there are a lot without paying attention to the real situation under question [LEISER, 1988].

Ideologically this state of affairs leans upon a methodological doctrine which is taking over a large part of the studies in Psychology Faculties:

this doctrine is carried out with an authentic methodical indoctrination which surprises most students. Only after they have registered do they realize that instead of studying psychology for its results at a theoretical level and its practices for the treatment of problems of human psychism they have to spend one class doing mathematics, two classes of statistics and two more classes of concepts of measurement and finally two experimental practices. So it is only

at the end of this obstacle race during which fifty per cent of my fellow students gave up the battle the chance to ask the following question comes back: what were my reasons in those days for deciding to study psychology?

To illustrate the mental disfiguration which is the result of this methodical indoctrination let's take the elemental reasoning which characterizes a statistical test. It is easy to demonstrate (as I have done in some papers) that in the last instance each statistical decision on a hypothesis has an inductive character whose rational explanation is not very satisfactory up to now. On the other hand the students learn to identify themselves completely with the critical rationalism which sustains the inadmissibility of induction in the sciences. Fortunately relative to the rejection of the null hypothesis the statistical terminology speaks of its falsification although in inductive terms. But it is enough to solve the contradiction and recover mental balance: confusing statistical falsification with Popper's falsification, the student may continue to believe in the authority of statistical tests and at the same time identify with critical rationalism as the basis of the scientific nature of his subject. Text books on statistics systematically share this misunderstanding (as I demonstrated in an analysis of the 6 most widely read text books in German [LEISER, 1982]).

However it is the minority who tolerate this kind of deformed psychology which has been reduced to ceremonial methods.

So this indoctrination of established psychology does not function satisfactorily but quite the opposite, it provokes a contrary movement. This is now manifested in the Universities although surreptitiously and even more so outside of them above all in the so-called psychotherapy market where one trend follows another and each one further and further from any methodical foundation. All these are aiming at the recuperation of an integral vision of human beings and methodically considered they tend to base the practice of psychology on an unlimited subjectivism at the expense of any objective foundation.

It is quite possible that the two antagonistic worlds of academic psychology bound to mathematics and the other which is distant to this ideal, may go together. As a result we find a fairly common type of psychology which preserves a certain mathematical identity in one part of the brain and the methodical pretensions that this implies, but in practice they end up abandoning these pretensions not without some remorse as they choose the most fashionable trend of the present day.

Now to end our expedition throughout the history of modern psychology, how shall we summarize the function of mathematics in it?

Firstly, (key words: psychophysics and associative psychology) the use of mathematical concepts followed what was known in classical physics: to represent and model well defined phenomenon and processes by means of a mathematical scheme as a functional relation. With regard to such well-defined objects we can speak of a tool-like function of mathematics within a superior theoretic context.

Lastly this theoretical context was abolished (key words:

behaviourism, black box) and mathematics became an ideological substitute to fill a conceptual vacuum. At the same time psychological research lost its well-defined objectives such as the concrete processes which constitute the psychic mechanism of the human being reducing this being to an abstract element in a survey test.

This use of statistics in psychology implies on the one hand the disappearance of concrete functions and structures in favour of a set of variables that characterizes the human population

in the statistical sense. On the other hand it means a schizophrenic approach to human psychism. For the researcher on annihilating these subjects, transforming them into elements as points of intersection of variables (let's say its point scoring on a scale of intelligence, another on neurosis another on extroversion etc), he acquits his own person of this destiny since he believes himself capable of managing and modelling the world of variables as he wishes in the laboratory.

To maintain this ideological basis of present day academic psychology and in order to relieve the afore mentioned schizophrenic sensations, a rigorous and well structured indoctrination is necessary that aims at the creation of absolute belief in the authority of mathematical thinking at the expense of its rationality. In effect mathematics lose their tool-like function only to be converted into something magical.

Now to finish, there are two singularities that characterize psychology in relation with T.S. Kuhn's analysis: the persistent lack of a paradigm and the role of mathematics which, instead of facilitating the elaboration

of a paradigm, have assumed an irrational function, namely that of an ideology which helps to maintain the state of things.

I wanted to draw your attention to this peculiarity of mathematics to ask the question what is it that distinguishes mathematics to assume such an ideological function?

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