The Economics of Pricing Radio Spectrum\textsuperscript{1}

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1 Introduction

Radio spectrum is a finite common access non-exhaustible resource that is used in many different applications, supporting both consumption and production activities. Over the last twenty years or so there has been a marked increase in the use of radio based services.\textsuperscript{1} Within households the use of radio has grown substantially, and the popularity of cellular telephony has led to a substantial increase in the demand for radio spectrum. In this paper we focus on the demand for, and the supply of, radio spectrum used to support production activities.

Unlike many other inputs used in production, in most countries it is not possible to trade radio spectrum frequency rights and obligations.\textsuperscript{2} Where markets are missing, this can give rise to inefficiency—especially where demand exceeds supply. Although an obvious remedy to the inefficiency arising from incompleteness is to permit trade of radio spectrum, this is viewed by many practitioners as a long-term solution.\textsuperscript{3} In the short-term therefore some radio authorities have introduced what is termed ‘incentive pricing’ to promote more efficient use of radio spectrum.\textsuperscript{4} Incentive pricing is intended to promote efficiency by establishing administered prices that reflect (estimated) opportunity costs of radio spectrum.

We consider in this paper the economic rationale for ‘incentive pricing’ of radio spectrum and look at the application of ‘Administrative Incentive Pricing’ (AIP) in the UK.

The structure of the paper is as follows. Section 2 discusses the allocation, assignment and characteristics of radio spectrum. In section 3 economic efficiency and the concept of a second-best optimum. Section 4 looks more closely at radio spectrum management policy and efficiency. It is argued that the attainment of efficiency in radio spectrum management means that the use of radio spectrum should accord with productive efficiency. Section 5

\textsuperscript{1}For example, see Cave (2002).

\textsuperscript{2}Markets for radio spectrum do exist in a few countries. In Australia spectrum trading was introduced in 199X, and trading of spectrum also occurs in New Zealand and Guatemala. Check other countries. Following the passage of the EU Regulatory Framework for Electronic Communications in 2002, member states of the EU may permit trading in spectrum (see article 9(3) of the Framework Directive, 2002/21/EC). The UK has introduced legislation permitting the trade in spectrum (Communications Act 2003, Section 168), and the regulator OFCOM is currently consulting on proposals for spectrum trading, Spectrum Trading Consultation, December 2003.

\textsuperscript{3}However, Kwerel and Williams (2002) have proposed an auction based method for a rapid transition to a market allocation of spectrum.

\textsuperscript{4}The Radiocommunications Agency in the UK first introduced incentive pricing for radio spectrum in 1998.
presents a simple model to illustrate the conditions necessary for productive efficiency. Section 6 describes the application of the Smith-NERA least-cost-alternative method and its relationship to efficiency. Section 7 concludes.

2 Allocation, assignment and characteristics of radio spectrum

Radio spectrum $S$ is a finite non-exhaustible common access resource extending between the frequencies 9 kHz and 3000 GHz.\textsuperscript{5} There are many public and private uses to which radio spectrum can be put, just as land can be used for many different purposes. International coordination has traditionally determined the uses to which radio spectrum is put and how much frequency is allocated to a use.\textsuperscript{6} For example, the frequency range 87.5 MHz to 108 MHz is allocated to FM sound broadcasting providing national, regional and local VHF radio services.\textsuperscript{7}

At a national level the use of radio spectrum $S$ in most countries is currently managed closely by government agencies rather than by market forces. The management of spectrum by government is usually predicated on protecting property rights, promoting the benefits associated with coordinating use and national security.

In many countries the primary tool of spectrum management by government is a licensing system. Although some frequency bands, such as frequencies around 2.4 GHz, are licensed exempt and akin to a ‘commons’, the majority of frequencies require users to hold a licence from government to access radio spectrum.\textsuperscript{8} Licensing effectively makes many frequencies in

\textsuperscript{5}At the present time, frequencies above 300 GHz have not been allocated to specific uses.

\textsuperscript{6}The United Nations organization the International Telecommunication Union oversees international coordination of radio spectrum via the World Radiocommunication Conference, see http://www.itu.int/ITU-R/conferences/wrc/index.asp.

\textsuperscript{7}By allocating this frequency range to VHF broadcasting, manufacturers of radio sets have designed equipment to operate around these frequencies. As a consequence, consumers possess equipment to receive VHF broadcasting within the frequency bands 87.5 MHz through to 108 MHz. While in principle it is possible to re-allocate these frequencies to another potentially higher-value use, there would in practice be a significant cost associated with reconfiguring or replacing the equipment used to receive sound broadcasting. In the future ‘smart radios’ may lessen the need for such rigid allocations of radio frequencies, see http://www.sdrforum.org/ for information about software defined radios.

\textsuperscript{8}Licences provide both rights and obligations. Rights provide for access to specified frequencies at particular times, and obligations may include constraints on system apparatus (such as constraints on power, physical size, etc.).
the radio spectrum have a *de facto* private good characteristic.\(^9\) The issuing of radio spectrum licences is known as the *assignment* of radio spectrum.

The allocation problem effectively involves the division of the radio spectrum \(S\) into a collection of *frequency bands* which are allocated to uses. Stating this formally, let \(s \in S\) be a frequency within the radio spectrum, then for an integer \(j \geq 1\) define \(b_j = (s_{j-1}, s_j) \subset S\), where \(0 < s_{j-1} < s_j\), as frequency band \(j\).\(^{10}\) If there are \(N\) possible uses for radio spectrum, the allocation problem may be solved by dividing \(S\) into \(N\) frequency bands \(b_j\) for \(j = 1, \ldots, N\).\(^{11}\)

There is little discretion in the short- to medium-term (which may be up to ten years) for individual national states to affect radio spectrum allocation. However, while fundamental changes to the way radio spectrum is allocated across uses is generally not feasible, changes to allocations at the margin are sometimes possible—giving scope for some policy intervention. There is usually much more discretion over the initial assignment of frequencies to users, and over the subsequent re-assignment of frequencies among users.

In making an initial assignment of frequency rights, several methods can be applied, including first-come first-served administrative methods and auctions.\(^{12}\)

### 3 Radio spectrum policy in the UK and efficiency

At the present time radio spectrum is not traded in the UK. Where there exists a charge for price for radio spectrum in a frequency band \(b_j\), call this \(a_j\), the price has been set administratively. A key issue for radio spectrum pricing policy is:

- What factors ought to determine the selection of an administrative prices \(a_j\)?

Until 1998 in the UK the criteria for setting the prices for radio spectrum were primarily cost recovery.\(^{13}\) Since 1998 increasing use has been made of AIP, where the primary criterion is economic efficiency.

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\(^9\)An alternative way to manage radio spectrum would be as a commons, in much the same way oceans are managed. This approach has been discussed by Faulhaber and Farber (2002).

\(^{10}\)The use of open intervals can be justified in terms of guardbands.

\(^{11}\)This is a simplification because in practice some uses share frequency bands.

\(^{12}\)On auctions see Cramton (2002) and...

\(^{13}\)The administrative charges \(a_j\) were set to recover the costs of spectrum management directly attributed to users, but also included an element to reflect common costs. In
In the Cave Review (2002) it is stated that AIP ought to be chosen in a way that encourages productive efficiency:

The fundamental mechanism by which the spectrum management regime could contribute to economic growth is through ensuring that users face continuing incentives towards more productive use of this resource. The review considers that these incentives should be financial and based on the opportunity cost of spectrum use. In this way, spectrum would be costed as any other input into the production process. Price signals about the cost of using spectrum would be disseminated throughout the economy. This information should enable dispersed economic agents to make their own judgements about their use of spectrum and the alternatives open to them to meet their organisational goals.14

A necessary condition for productive efficiency is the equalisation of the marginal rate of technical substitution between inputs across sectors (and across firms within a sector).15 This suggests that for radio spectrum use to satisfy productive efficiency the marginal rate of technical substitution between radio spectrum and another substitutable input should be equal across firms in the same sector, and across firms using these inputs in different sectors.

The justification for ensuring that the use of radio spectrum satisfies productive efficiency can be found in Diamond and Mirrlees (1971). Diamond-Mirrlees state that a policy maker choosing taxes in a second-best setting should not tax the use of inputs. It is further stated that the use of inputs in a competitive economy should satisfy conditions necessary for productive efficiency if a second best outcome is to be achieved.

**Proposition 1** If competitive markets operate pervasively in an economy and regulation mimics market outcomes otherwise, the Diamond-Mirrlees result suggests that government should only tax final goods and services if a

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14 Para. 22, page 7, Cave (2002). Emphasis added. Opportunity cost is defined in the Cave Review as: “the value of an asset or resource in the next best alternative that is foregone by virtue of its actual use.”

15 This implicitly assumes that production technologies give rise to convex continuously differentiable isoquants.
second-best welfare maximum outcome is to be achieved. This implies that
the setting of AIP should be consistent with productive efficiency, and that
AIP should reflect opportunity costs. When this occurs, a second-best welfare
maximum could be achieved.

Setting the price for spectrum so that productive efficiency is promoted,
both within sectors and across sectors, is therefore desirable.

4 Productive efficiency and the use of radio
spectrum

A simple two-sector, two-firm framework is presented to outline in slightly
more detail the concept of productive efficiency in the context of radio spec-
trum use.

4.1 Assumptions

There are two sectors (uses) 1 and 2 in the economy, and production in each
sector requires radio spectrum. There is a finite amount of radio spectrum
$S = 1$, where $0 < y < 1$ is the amount allocated to use 1 and $1 - y$ is the
amount allocated to use 2.

There is one firm operating in each sector 1 and 2. Each firm produces
good $q_1$ and $q_2$ respectively using radio spectrum $s$ and another input $x$
(where $x$ may refer to base stations or labour, etc.). The price of radio
spectrum is $a > 0$ per unit. There is a competitive market for the other
input $x$, which has a price $w > 0$.

The production technologies of the two firms are $q_1 = f(x_1, s_1)$ and $q_2 =
g(x_2, s_2)$. Output $q_i$ increases as more of each input is used;\footnote{This
assumes away fixed proportion Leontief type technologies, which may describe
some use areas of radio spectrum.} that is the marginal products are positive ($f_x, f_s, g_x, g_s > 0$),\footnote{Throughout $f_v$ refers to the partial derivative of the function $f$ with respect to variable $v$.} and there are diminishing
returns: $(f_{xx}, f_{ss}, g_{xx}, g_{ss} < 0)$.

Both $q_1$ and $q_2$ are sold on competitive markets and the firms take prices
$p_1, p_2 > 0$ as given.
4.2 Behaviour

Every firm maximizes profit, where $\Pi_i$ denotes the profit of the firm in sector $i = 1, 2$. The firm in sector 1 achieves this objective by choosing an appropriate configuration of inputs, taking as given prices, its production technology and spectrum constraints:

$$\max \Pi_1(x_1, s_1) = p_1 q_1 - wx_1 - as_1$$

s.t. $q_1 = f(x_1, s_1)$ \hspace{1cm} (1)

$s_1 \leq y$.

The solution to (1) is:

$$p_1 f_x = w$$

$$p_1 f_s = a, \text{ or}$$

$$p_1 f_x = w$$

$$p_1 f_s > a.$$ \hspace{1cm} (3)

Where (2) is the solution, the spectrum constraint is not binding. However, for the solution shown in (3) additional spectrum would be used by firm 1 as its profit could increase—indicating a binding spectrum constraint.

Analogous conditions to (2) or (3) would also hold in the other sector 2. The following table illustrates the range of possibilities and describes their efficiency properties:

<table>
<thead>
<tr>
<th>Case</th>
<th>$p_1 f_s = p_2 g_s = a$</th>
<th>$s_1^* = y, \ s_2^* = 1 - y$</th>
<th>Efficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$p_1 f_s = a, \ p_2 g_s = a$</td>
<td>$s_1^* &lt; y, \ s_2^* &lt; 1 - y$</td>
<td>Inefficient</td>
</tr>
<tr>
<td>(ii)</td>
<td>$p_1 f_s &gt; a, \ p_2 g_s = a$</td>
<td>$s_1^* = y, \ s_2^* = 1 - y$</td>
<td>Inefficient</td>
</tr>
<tr>
<td>(iii)</td>
<td>$p_1 f_s = a, \ p_2 g_s &gt; a$</td>
<td>$s_1^* = y, \ s_2^* = 1 - y$</td>
<td>Inefficient</td>
</tr>
<tr>
<td>(iv)</td>
<td>$p_1 f_s &gt; a, \ p_2 g_s &gt; a$</td>
<td>$s_1^* = y, \ s_2^* = 1 - y$</td>
<td>Inefficient</td>
</tr>
</tbody>
</table>

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18 In an extension we shall consider the case where a spectrum user is not a profit maximizing firm. For example, there are many public sector users of spectrum who seek to minimise the costs of producing an output level.
Case (i) is efficient because the two conditions necessary for productive efficiency are satisfied. These are:

1. The marginal rates of technical substitution across the two sectors are equalised, AND

2. All available spectrum in the economy is used.

Note that case (i) implies that the correct price for radio spectrum is at the point where the marginal revenue products with respect to spectrum are equalised. Hence, the opportunity cost of spectrum in sector 1 is equal to the value foregone by using spectrum in sector 1 rather than in sector 2—and similarly the converse holds.

In every other case inefficiency occurs. In case (ii) the radio spectrum price is too high (it is above the opportunity cost). In case (iii), although radio spectrum is fully utilised, productive efficiency does not hold because too much spectrum is allocated to sector 2. In other words, by reallocating spectrum the same outputs could be produced and some spectrum could be freed-up, implying that outputs overall could be made greater. Cases (iv) and (v) are qualitatively similar to case (iii).

From a policy perspective, there is scope to improve productive efficiency where the inequality:

\[ p_i f_s > a \]  

holds. Even where marginal revenue products are observed equal to the price of spectrum, if some spectrum lies unused productive efficiency gains would be feasible.

In the next section the Smith-NERA framework is described. The application of the Smith-NERA method permits the calculation of AIP that promote productive efficiency.

## 5 The Smith-NERA least cost alternative method

The Smith-NERA method works by identifying the rate of technical substitution between radio spectrum and another input such that the quantity of output produced by a firm is assumed not to change. One of the strong appeals of the Smith-NERA least-cost alternative method is its simplicity. Furthermore, applied correctly it permits the promotion of productive efficiency. In this section the method is illustrated and its application demonstrated.

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19The Smith-NERA method is a way of estimating the slope of an isoquant.
5.1 Applying the Smith-NERA method

Suppose firm \( i \) in sector \( j \) which uses spectrum \( s^j_i \) and another input (say base stations \( b^j_i \)) produces output \( q^j_i \). The firm’s output can be expressed as:

\[
q^j_i = f(b^j_i, s^j_i).
\] (5)

Assume the firm is a profit maximiser and hence minimises costs. If a unit of spectrum \( \Delta s \) is added to or subtracted from \( s^j_i \), a compensating change could be made in the amount of the other input \( b^j_i \) such that total output is unchanged at \( q^j_i \). By doing this the rate of technical substitution between the two inputs can be assessed. For a \( \Delta s = 1 \) there would be an implied change \( \Delta b \), and where the latter is multiplied by its price (which is determined on a competitive market) this allows for a money representation of the rate of substitution (where money is the numeraire). By applying the same procedure in other sectors, comparisons can be made across sectors using the common unit money (which means comparisons can be made across sectors where different inputs substitute for spectrum).

In Table 1 below we present a hypothetical example illustrating the Smith-NERA least cost alternative method. The values in the cells are calculated as described in the preceding paragraph. Hence, 100 in use I, frequency band \( a \) is the value, expressed in money terms using the least-cost-alternative input, of the marginal unit of spectrum. For example, a unit of spectrum may be worth four base station which each have a price 25.

The values in the other cells also represent the value of a marginal unit of spectrum. For productive efficiency to be satisfied, spectrum ought to be allocated across uses so that these values are identical. It can be seen in the table that they are not equal. The example is an illustration of productive inefficiency.\(^{20}\)

<table>
<thead>
<tr>
<th>Frequency Bands</th>
<th>Non radio spectrum input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses</td>
<td>( a )</td>
</tr>
<tr>
<td>I</td>
<td>100</td>
</tr>
<tr>
<td>II</td>
<td>35</td>
</tr>
<tr>
<td>III</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1: Hypothetical example

\(^{20}\)If re-allocation of radio spectrum were not possible, then the values in Table 1 could be compatible with productive efficiency. In particular, if firms in each use area differ and the values in the cells represent averages, then productive efficiency would be achieved in each use area if the price of spectrum were set equal to the identified opportunity cost.
Further application of the Smith-NERA method leads to recommended prices for radio spectrum that are consistent with productive efficiency.

Consider the values in the row associated with Use I. The marginal value of frequency band \( a \) in Use I is 100 and the marginal value of frequency band \( b \) in Use I is 75. Note that frequency band \( b \) is an imperfect substitute for frequency band \( a \) in Use I. However, the marginal value of frequency band \( b \) in Use II is 60. Society would be better off therefore if some of frequency band \( b \) were re-allocated to Use I. This is because a marginal unit of frequency band \( b \) applied to Use I could produce the same output in Use I while freeing up enough resources to compensate Use II (and hence maintain a constant output in Use II) and provide some extra resources for additional production in the economy.

The above can be stated in terms of opportunity costs. The opportunity cost of frequency band \( b \) spectrum in Use II is 75, the foregone saving in terms of least-cost-alternatives that would arise if the frequency band were used in Use I (the next best alternative). By expressing the value of marginal spectrum in terms of opportunity cost, it is possible to address the issue of pricing radio spectrum.

What should the administrative price be for radio spectrum frequency band \( b \)? This is determined by permitting variation in the frequency bands allocated to the three uses. It is clear that more of frequency band \( b \) ought to be allocated to Use I, and more frequency band \( c \) should be allocated to Use II. In Tables 2 and 3 below we present illustrations of the effect of re-allocating spectrum in this way.

<table>
<thead>
<tr>
<th>Uses</th>
<th>Frequency Bands</th>
<th>Non radio spectrum input</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>87 64 0</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>32 64 25</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>13 12 25</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 2: Marginal values after reallocation of spectrum

<table>
<thead>
<tr>
<th>Uses</th>
<th>Frequency Bands</th>
<th>Non radio spectrum input</th>
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<tr>
<td>I</td>
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<td>32 56 25</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>13 12 25</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 3: Marginal values after reallocation of spectrum
6 Conclusion

The Smith-NERA least-cost-alternative method provides a relatively simple tool for identifying value ranges which can be used to guide the setting of administrative prices for radio spectrum. The prices established accord with the principle of opportunity cost and are consistent with the objective of achieving efficiency (in particular, productive efficiency). Furthermore, the Smith-NERA method can be extended to account for the relationship between radio spectrum and quality, though in practice this is likely to be informationally onerous.

Although one of the appeals of the Smith-NERA method is its simplicity, it may not always be possible to apply it in every circumstance. For example, where a production technology is all or nothing (as may be the case with some broadcasting), it is questionable whether such ‘marginal’ analysis could be applied. However, even in these circumstances it ought to be possible to construct opportunity cost estimates of spectrum—and hence establish guides for spectrum prices consistent with productive efficiency.
References


