Problems on Algebra III

Winter 2021

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Problem Set 2

Due: Tuesday, November 9, 2021, 2pm

Exercise 1 (The spectrum of $\mathbb{R}[t]$; 10 points).
Describe the spectrum of $\mathbb{R}[t]$ as a topological space and relate it to $\mathbb{R}$ and $\mathbb{C}$.

Exercise 2 (The spectrum of $\mathbb{F}_p[t]$; 5+15 points).
Let p be a prime number and $\mathbb{F}_p := \mathbb{Z}/\langle p\rangle$ the field with p elements.
a) Which fields do occur as residue fields at a point $x \in \mathrm{Spec}(\mathbb{F}_p[t])$?
b) Given a residue field k, how many points are there in $\mathrm{Spec}(\mathbb{F}_p[t])$ whose residue field is isomorphic to k?
Hint. The solution is not straightforward and might involve some research into the literature.

Exercise 3 (The spectrum of $\mathbb{Z}[t]$; 10 points).
Describe the spectrum of $\mathbb{Z}[t]$ and the map to $\mathrm{Spec}(\mathbb{Z})$ that is associated with the homomorphism $\mathbb{Z} \longrightarrow \mathbb{Z}[t]$.

Exercise 4 (Infinite products of fields; 10 bonus points).
Let k be a field and $R := \prod_{k=0}^{\infty} k$. Describe $\mathrm{Spec}(R)$.