On actions of non-reductive groups with a graded unipotent radical on affine varieties

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Abstract

As a motivation, we consider a moduli functor which is given by a fibre of the moduli functor for quiver sheaves. This moduli problem generalises the set up considered by Drezet and Trautmann [DT]. For a fixed quiver and a tuple of decomposable sheaves indexed by the vertex set of the quiver, we consider homomorphisms between these sheaves given by the arrows in the quiver. This moduli problem then involves constructing a quotient of a product of automorphism groups of sheaves acting on an affine space. The automorphism groups of the sheaves are in general non-reductive, and in fact their unipotent radical is (internally) graded in the sense that there is a central 1-parameter subgroup in the reductive Levi factor which acts on the Lie algebra of the unipotent radical with strictly positive weights.

For actions of such internally graded non-reductive groups H on projective varieties, the results of [BDHK] are strongest (for example, one can obtain a Hilbert-Mumford type description of semistability and construct (quasi)-projective quotients). In fact more generally, one should hope to have similar results for projective over affine varieties, as in the reductive case this is the most general set up for which there is a Hilbert-Mumford criterion.

The set up of the above moduli functor naturally leads to the problem of establishing similar results for actions of a non-reductive group with internally graded unipotent radical on an affine space with respect to a character, which should extend King's work [K] from the reductive setting to this non-reductive setting.

References
[BDHK] G. Berczi, B. Doran, T. Hawes and F. Kirwan "Projective completions of graded unipotent quotients."
[DT] J.-M. Drézet and G. Trautmann "Moduli spaces of decomposable morphisms of sheaves and quotients by non-reductive groups."
[K] A. King "Moduli of representations of finite dimensional algebras."