The harmonic Green and Neumann functions of domains bounded by two intersecting circular arcs

Abstract

To obtain explicit solutions to the Dirichlet and Neumann boundary value problems of the complex Poisson equation over a domain in $\mathbb{C}$, we usually need to know the harmonic Green function and the harmonic Neumann function of the domain. They are two fundamental solutions of the complex Laplace operator $\partial_z\partial_{\bar{z}}$ that satisfy the corresponding boundary properties. Unfortunately, the expressions f of the harmonic Green and Neumann functions are known only in a few domains. It is often difficult to calculate the harmonic Green and Neumann functions for a given domain. For domains whose boundaries consist of circular arcs or lines, the so-called parqueting-reflection principle is applied to help determine the harmonic Green and Neumann functions. It has been verified to be feasible for certain such domains, such as discs, half-discs, cones, sectors, strips, half-strips, concentric rings, hyperbolic strips, etc. (see [1-8]).

In this talk, we will introduce the parqueting-reflection principle and show that this principle also works for a particular type of domain, which is bounded by two circular intersecting arcs with an angle π/n for a positive integer n. We will see how this principle helps construct the explicit formulas of the harmonic Green and Neumann functions for these domains. Then we will deal with the Dirichlet and Neumann boundary value problems of the complex Poisson equation, and give explicit solutions by using the Green representation formula and the Neumann representation formula.

References

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