The Triangle Equation, also known as the Yang-Baxter equation, was discovered around 1970 by Yang (in quantum field theory) and Baxter (in statistical mechanics). It now plays an important role for quantum groups and was used in the construction of invariants of knots.

In this talk I shall start with two elementary topics: the arithmetic-geometric mean and braid groups. The arithmetic-geometric mean is closely related to elliptic integrals (Lagrange and Gauss). From there it is only one step to elliptic functions. Representations of braid groups will lead us to the Triangle Equation. Its classical limit is known as the classical Yang-Baxter equation.

I shall explain how vector bundles on elliptic curves give rise to elliptic solutions of this equation. The degeneration of such solutions into trigonometric or rational solutions can be explained geometrically and is linked to the degeneration of an elliptic curve into a nodal or cuspidal rational curve.