Advanced Statistical Physics
Problem Set 8

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available at http://userpage.fu-berlin.de/psilvest/

Problem 8.1 The averaged kinetic energy of a molecule of a dilute monoatomic gas in a vessel is $3k_BT/2$. Find the averaged kinetic energy of a molecule hitting the wall of the vessel. **[2p]**

Problem 8.2 Estimate the number of molecules in the lecture room with energies $E > 1\text{eV}$, $E > 2\text{eV}$, $E > 3\text{eV}$, $E > 4\text{eV}$. If the number of such molecules happens to be smaller than 1, estimate the time after which such a molecule will appear? (Hint: Suppose such a molecule did appear. You may easily estimate, how long it would live.) **[2p]**

Problem 8.3 A two level system has a nondegenerate lower level, $\varepsilon_0 = 0$, and an N-fold degenerate upper level, ε_1. Find the heat capacity $c(T)$ of the system. Consider in more details the case $N \gg 1$ (even $\ln N \gg 1$). In this case, find the position and the shape of the maximum of $c(T)$. What is the physical reason for the emergence of a sharp maximum in the heat capacity in case of a strongly degenerate upper level. (Hint: Write the temperature as $T = T_0 + \tau$, with $e^{\frac{\varepsilon_1}{k_B T_0}} = N$, and consider the region $\tau \ll T_0$.) **[2p]**

Problem 8.4 The spectrum of the hydrogen atom contains an infinite number of discrete energy levels, $\varepsilon_n = -Ry/2n^2 = -13.6\text{eV}/n^2$. This should mean that the partition function of the atom is infinite. How can statistical physics live with that? **[2p]**

Problem 8.5 A molecule of NH_3 may exist in two quantum mechanical states with very close energies ε_1 and ε_2 ($\varepsilon_2 - \varepsilon_1$ is small compared with the excitation energy of the first rotational level). Find the partition function $Z(N, T, V)$ of the dilute NH_3 gas at low temperatures. Find the energy and the heat capacity c_V. Calculate the temperature dependance of the probability to find a particular molecule in one of the states $\varepsilon_1, \varepsilon_2$. **[2p]**