Advanced Statistical Physics
Problem Set 7

due: 08.12.2010
available at http://userpage.fu-berlin.de/psilvest/

Problem 7.1 Extension of problem 6.2 from the previous set: You probably already noticed that in this problem you were calculating the entropy of the ensemble of M harmonic oscillators

$$\mathcal{H}(\{n_i\}) = \sum_{i=1}^{M} \hbar\omega(n_i + \frac{1}{2}), \tag{1}$$

with the energy

$$E = \hbar\omega(N + \frac{M}{2}). \tag{2}$$

(a) Continue the same calculation and find the probability $p(n)$ that a particular oscillator is in its n-th level.

(b) Find $p(n)$ from the Canonical ensemble. Compare results for Canonical and Microcanonical ensembles.

(c) Find the average energy ε per oscillator at a given temperature T. Use the Canonical ensemble and formulas from the lecture. **[2p]**

Problem 7.2 The Canonical ensemble: Show that if the temperature is uniform, the pressure of a gas in a uniform magnetic field decreases with height according to $P(z) = P(0)\exp(-mgz/k_B T)$, where m is the mass of the gas molecule. **[2p]**

Problem 7.3 Rubber molecule: A one-dimensional polymer molecule consists of N ($N \gg 1$) elements of length a. Let the distance between the ends of the molecule be x (see figure). Find the entropy of the molecule as a function of x and obtain the relation between the temperature of the chain and the force(tension) which is necessary to maintain the distance x at a temperature T, assuming the joints can turn freely.

Is not it amazing that a molecule with zero internal energy can cause a nontrivial tension?

Small hint: You may apply your knowledge of Statistical physics to find the Free energy and then use the standard machinery of Thermodynamics. **[2p]**

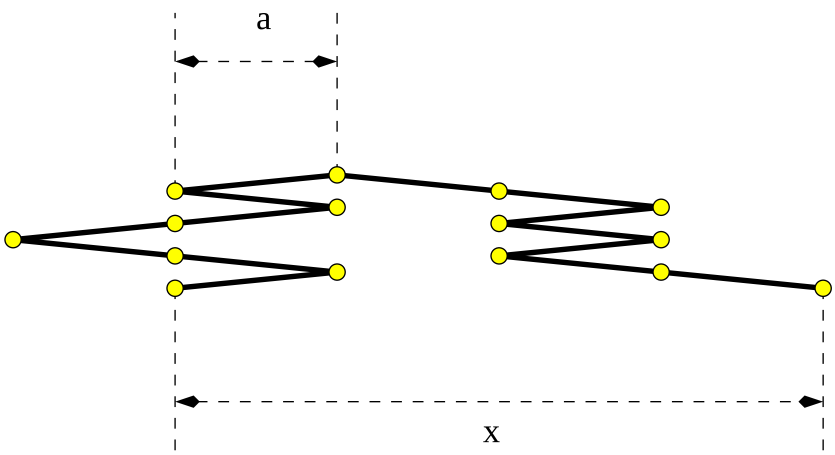

Figure 1:

Problem 7.4 Molecular adsorption: N diatomic molecules are stuck on a metal surface of a square symmetry. Each molecule can either lie flat on the surface, in which case it has to be aligned to one of two directions, x and y, or it can stand up along the z direction. There is an energy cost of $\varepsilon > 0$ associated with a molecule standing up and zero energy for molecules lying flat along x or y directions.

(a) How many microstates have the smallest value of energy? What is the largest microstate energy?

(b) What is the largest positive energy at any positive temperature? **[2p]**

You (almost) do not need to do any calculation to answer the previous 2 questions. Now use the Microcanonical ensemble:

(c) For *microcanonical* macrostates of energy E, calculate the number of states $\Omega(E, N)$, and the entropy $S(E, N)$.

(d) Calculate the heat capacity $C(T)$ and sketch it.

(e) What is the probability that a specific molecule is standing up? **[2p]**