Advanced Statistical Physics
Problem Set 4

due: 17.11.2010

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1. Use the proper Maxwell relation to show that

$$\frac{\partial(P,V)}{\partial(T,S)} = 1. \tag{1}$$

This means that any cycle has the same area if drawn on P,V or T,S planes. You may start from $dE = TdS - PdV$. [2p]

2. Use Jacobians and the previous problem to show that

$$C_P - C_V = -T\left[\left.\frac{\partial V}{\partial T}\right|_P\right]^2 \left.\frac{\partial P}{\partial V}\right|_T. \tag{2}$$

Since, as we learned on the lecture, always $\partial P/\partial V|_T < 0$ for thermodynamically stable systems, this result implies that $C_P > C_V$.

Prove the relation for compressibilities

$$\left.\frac{\partial V}{\partial P}\right|_S = \frac{C_V}{C_P}\left.\frac{\partial V}{\partial P}\right|_T. \tag{3}$$

Since $C_P > C_V$, the adiabatic compressibility is always smaller than the isothermic one. [2p]

3. Clapeyron Clausius formula: Find the equilibrium radius of a water droplet in contact with over-saturated vapor.

The temperature is T. The equilibrium pressure for a contact *vapor-flat water surface* for this temperature is P_0. The pressure of over-saturated vapor is $P = P_0 + \delta P$ and the surface tension for the water is $\mathcal{S}$. (You will need to invent your own Clapeyron Clausius formula. Don't forget problem 4(a) from Problem Set 1.)[2p]

Consider the stability of such equilibrium. [0p]

4. Thermodynamics of the superconducting transition: The thermodynamics of a metal in a magnetic field is described by the Free energy per unit volume, $\mathcal{F}$, $d\mathcal{F} = -SdT + \frac{1}{4\pi}HdB$. Here H is the magnetic field and B is the magnetic induction, $B = H + 4\pi M = \mu H$. Since the quantity which is controlled by external magnets is the induction H, one usually introduces the analogue of the Gibbs Free energy $\mathcal{F}^H = \mathcal{F} - \frac{HB}{4\pi}$, $d\mathcal{F}^H = -SdT - \frac{1}{4\pi}BdH$. For a constant magnetic permeability μ this gives $\mathcal{F}^H = \mathcal{F}^{H=0} - \mu\frac{H^2}{4\pi}$.

At low temperatures many metals can exist in two (meta)stable states: a normal state (n) and a superconducting state (s). For the normal metal $\mu = 1$, while for the superconductor the famous Meissner effect implies $\mu = 0$ (no magnetic induction can penetrate into the superconductor). Thus we find

$$\mathcal{F}_n^H = \mathcal{F}_n^{H=0} - \frac{H^2}{4\pi}\ , \ \mathcal{F}_s^H = \mathcal{F}_s^{H=0}. \tag{4}$$

From experiment it is known, that superconductivity may be destroyed by applying a magnetic field larger than some critical field $H_C(T)$. Translated into thermodynamic language this means $\mathcal{F}_n^H < \mathcal{F}_s^H$ at $H > H_C$, and $\mathcal{F}_n^H > \mathcal{F}_s^H$ at $H < H_C$. This allows us to relate the difference of the Free energies of normal and superconducting phases in the absence of a magnetic field

$$\mathcal{F}_n^0 - \mathcal{F}_s^0 = \frac{H_C^2(T)}{8\pi}. \tag{5}$$

(The function $H_C(T)$ is well approximated by $H_C = H_C(0)(1 - T^2/T_C^2)$, but you may use only that H_C vanishes at some critical temperature $H_C(T_C) = 0$.)

(a) Show from the third law of thermodynamics that

$$\left.\frac{dH_C}{dT}\right|_{T=0} = 0. \tag{6}$$

(b) Find the latent heat ($q = T\Delta S$) for the $n - s$ transition. Show that $q = 0$ at the critical temperature, i.e. the transition here is of the second order.

(c) Find the difference of heat capacities $c_s - c_n$. Show that the difference changes sign at some temperature $T < T_C$. [2p]

5. Probabilities: Benjamin's mother lives in Krumme Lanke and his girlfriend rents an apartment in Schöneberg (all names changed). Every evening Benjamin leaves FU to the Dahlem-Dorf U-bahn station and takes the first train which he sees. He believes that in this way he would have dinner in about half of the evenings with his girlfriend and the second half of evenings the with his mother. However, Benjamin's mother complains that she saw him only 4 times in the last 20 days. But Benjamin is not cheating. How could that happen? [2p]