Advanced Statistical Physics
Problem Set 2

due: 3.11.2010

Problem 2.1 Which of the following may correspond to the internal energy E of a "hypothetical" thermodynamic system described by the coordinates x and y (and why of course):

$$(a)\ dE = \frac{-ydx}{x^2+y^2} + \frac{xdy}{x^2+y^2} \tag{1}$$

$$(b)\ dE = (y-x^2)dx + (x+y^2)dy \tag{2}$$

$$(c)\ dE = (2y^2-3x)dx + 4xydy. \tag{3}$$

Find the energy explicitly for these cases. **[2p]**

Problem 2.2 For the Carnot cycle considered on the lecture find a work done and a heat transferred at each segment. The cycle is performed with the ideal dilute mono-atomic gas whose internal energy and equation of state are $E = \frac{3}{2}Nk_BT$ and $PV = Nk_BT$. To define uniquely the cycle you need to know the temperatures T_H(source), T_C(sink) and for example two volumes V_1, V_2 between which the low temperature adiabatic process operates. Find the efficiency of the cycle and prove the equivalence of the ideal gas temperature and the thermodynamic temperature. **[2p]**

To do with the tutor: More general ideal gas has the equation of motion $PV = Nk_BT$, but its internal energy may be some more complicated function of temperature (and only temperature) $E = E(T)$. Prove the equivalence of the ideal gas and thermodynamic temperature scales in this case. **[0p]**

Problem 2.3 Consider an engine operating the cycle on the T, S plane

$$\left(\frac{S-S_0}{\Delta S}\right)^2 + \left(\frac{T-T_0}{\Delta T}\right)^2 = 1. \tag{4}$$

Find an efficiency of the cycle. Draw on the T, S plane the Carnot cycle enclosing our cycle. Prove that any such Carnot cycle is more effective than the cycle Eq. (4). **[2p]**

Is not the Carnot theorem obvious on the T, S plane?

Problem 2.4 Consider a capacitor (capacity $C = \varepsilon C_0$) filled with the dielectric whose dielectric constant depends on temperature, $\varepsilon = \varepsilon(T)$. The capacitor is connected to a heat bath (i.e. is kept at a constant temperature) and is slowly charged to the voltage V. Find the amount of heat absorbed by the capacitor. **[2p]**

Hints: Equation of state for the capacitor with a charge q is simply

$$V = \frac{q}{C(T)}. \tag{5}$$

Thermodynamic variables now are "force" $J = V$ and "displacement" $x = q$. You need first to choose and calculate a change of a proper thermodynamic potential.

Problem 2.5 A force J acting on an elastic filament is related to displacement x via

$$J = ax - bT + cTx, \tag{6}$$

where a, b, c are constants. Furthermore the heat capacity at constant displacement is $C_x = A(x)T$. Do at least one from the following:

(a) Calculate $\partial S/\partial x|_T$.

(b) Show that A should be independent of x, i.e. $dA/dx = 0$.

(c) Find $S(T, x) - S(0, 0)$.

[2p]