Advanced Statistical Physics
Problem Set 12

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available at http://userpage.fu-berlin.de/psilvest/

Problem 12.1 At what pressure the density of hydrogen will be 10kg/cm^3? How much would this answer change if the temperature was 3000K and not 300K? **[2p]**

Problem 12.2 Stellar dynamics continued. For a White Dwarf somewhat heavier than our Sun the reaction $e + p \to n + \nu$ may start. Massless and noninteracting neutrinos leave the star and the process leads effectively to the conversion of electrons and protons into neutrons. Thus a Neutron Star is born. Estimate the radius of the Neutron Star having the mass of the Sun. Use numbers from problem **11.5** **[2p]**

Problem 12.3 If the Neutron star is squeezed further, the neutron gas becomes relativistic. Investigate the fate of an ultrarelativistic star, where the energy of neutrons is given by

$$\varepsilon = \sqrt{m^2c^4 + p^2c^2} \approx pc. \tag{1}$$

How heavy should the initial star be to be able to give birth to the Black Hole? **[2p]**

Problem 12.4 Find the average squared displacement of the atom in a solid $\langle \vec{u}^2 \rangle$ in the Debye model. If you can not calculate the final integral, analyze the high/low temperature limits. The mass of the atom is m, sound velocity v, the crystal has N atoms in the volume V. **[2p]**

At what temperature the amplitude of the lattice vibrations will be comparable to the interatomic distance? **[0p]**

You start with writing the displacement of the atom i as a sum over displacements corresponding to individual phonon modes (wave vector $\vec{k}$)

$$\vec{u}_i = \sum_k \vec{u}_i^k. \tag{2}$$

Then you notice that due to the translational symmetry the squared displacement in each mode $|\vec{u}_i^k|^2$ does not depend on the position of the atom i. Finally you know from Quantum mechanics that the energy of an oscillator is equally split between the (average) kinetic and potential energy. Thus the thermal average ($\langle \cdots \rangle$) of $|\vec{u}_i^k|^2$ is related to the thermal average of the single phonon mode oscillator energy ε_k

$$\frac{Nm\omega_k^2 \langle |\vec{u}_i^k|^2 \rangle}{2} = \frac{1}{2} \langle \varepsilon_k \rangle. \tag{3}$$

Problem 12.5 Consider the (3-dimensional) Bose gas in the potential $U = m\omega^2 \vec{r}^2/2$. Find the temperature of the Bose-Einstein condensation and the discontinuity of the heat capacity at this temperature. Number of particles is N. You may need functions f_m^+ and ξ_m defined in Kardar's book. **[2p]**