Advanced Statistical Physics
Problem Set 11

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Problem 11.1 Find the Fermi energy (electron Volts), Fermi wave length (centimeters) and pressure (atmospheres) of gas of conduction electrons in copper. Assume that every atom provides one conduction electron and the effective mass in the conduction band coincides with the electron mass. The atomic mass and the density of copper are $A = 64$ and $\rho \approx 9g/cm^3$. [**2p**]

Problem 11.2 Some string Theorists believe that life would be a lot easier in 26(25+1) dimensional world. Find the heat capacity of a degenerate gas of (non-relativistic) electrons in a 25-dimensional metal. The effective electron mass is m, the density of electrons n, the temperature $T \ll E_F$ and spin $s = 1/2$. [**2p**]

Problem 11.3 Making problem **10.5** more realistic. Electrons in the conduction band of a semiconductor have a dispersion

$$\varepsilon = \frac{p^2}{2m^*}. \tag{1}$$

However, in a pure semiconductor at reasonable temperatures the density of electrons is negligibly small. In order to effectively add electrons to the conduction band, one adds donor impurities (density n_d) to the semiconductor. Each impurity brings exactly one mobile electron to the system.

Consider the model in which each donor creates a single energy level $\varepsilon_0 < 0$. However now this level may be occupied buy 0 or 1 spinful noninteracting electrons. Due to their strong repulsion, 2 electrons (even with opposite spins) can never occupy the same impurity. Find the averaged number of electrons on the impurity as a function of temperature.

In this problem, to describe the electrons on the impurities, you can not use the known formulas for occupation numbers for Fermi/Bose statistics, but should start again from the grand canonical ensemble and derive your own formula for the averaged occupation number. As always, the density of impurities is sufficiently small so that the electron gas in the conduction band obeys Maxwell statistics. [**2p**]

Problem 11.4 "Baby" Thomas-Fermi model. Most of the electrons in an atom with large $Z \gg 1$ move close to the Nucleus. In the Thomas-Fermi model these electrons are treated as a degenerate Fermi gas at zero temperature. Neglect the interaction between electrons and estimate the radius R of the electron cloud around the nucleus.

Hint: You need to minimize the sum of the total kinetic and potential energies of the cloud. Since you are asked only to give an "estimate", it is enough also to only estimate (up to a numerical factor) both contributions to the energy. **[2p]**

Problem 11.5 Radius of a White Dwarf. After a normal star has burned out most of its Hydrogen (creating Helium), the nuclear fusion process stops and the star starts to cool down and to shrink. For a not too heavy star this shrinking will stop because of the increasing kinetic energy of the degenerate electron (Fermi)gas in the star. Estimate the radius of the White Dwarf formed after the death of our Sun, mass $M \approx 2 \times 10^{33}$gram.

Again you need to compare the gain in the potential energy of the shrinking gravitating body and the increase of kinetic energy of the dense electron gas. The gravitational constant $G \approx 6.674 \times 10^{-8} \mathrm{cm}^3 \mathrm{g}^{-1} \mathrm{s}^{-2}$. **[2p]**