Advanced Statistical Physics
Problem Set 10

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available at http://userpage.fu-berlin.de/psilvest/

Problem 10.1 Lattice vibrations. Cooling of Hydrogen leads to condensation at temperatures below $T <$20K and then solidification at $T <$14K. Similar would be the behavior of Deuterium (with somewhat different temperatures). Find the ratio of sound velocities v_H/v_D in solid Hydrogen and solid Deuterium. **[2p]**

Problem 10.2 Prove that the density matrix of a pure system, ρ, satisfies the relation

$$\rho^2 = \rho. \tag{1}$$

[2p]

Problem 10.3 Consider a spin described by the Hamiltonian

$$H = \vec{B} \cdot \vec{\sigma}, \tag{2}$$

where $\vec{B}$ is a constant vector (magnetic field). Find the equilibrium density matrix for the spin at temperature T (Canonical ensemble). **[2p]**

Problem 10.4 Canonical density matrix for noninteracting identical particles. In the lecture (following the Kardar book) we considered the partition function for many noninteracting identical particles in the high temperature limit. We were able to find the leading contribution and the correction due to a single permutation of 2 particles. Try to consider the next correction, caused by two permutations.

Problem: The single-permutation correction to the partition function turns out to be of the relative order

$$\frac{\delta Z_1}{Z_0} \sim N^2 \frac{\lambda^3}{V}. \tag{3}$$

Taking into account this correction led to the correction to the Free energy

$$\delta F_1 \sim k_B T \frac{N^2}{V}, \tag{4}$$

which, as it should be, is an extensive function.

Simple counting of the powers of N and V suggests a 2-permutation correction of the form

$$\frac{\delta Z_2}{Z_0} \sim N^4 \frac{\lambda^6}{V^2} \ , \ \delta F_2 \propto k_B T \frac{N^4}{V^2}. \tag{5}$$

This result, if it is correct, means that δF_2 is not an extensive quantity. Resolve the paradox. **[2p]**

Problem 10.5 Donors and electrons in a semiconductor. Electrons in the conduction band of a semiconductor have a dispersion

$$\varepsilon = \frac{p^2}{2m^*}. \tag{6}$$

However, in a pure semiconductor at reasonable temperatures the density of electrons is negligibly small. In order to effectively add electrons to the conduction band, one adds donor impurities (density n_d) to the semiconductor. Consider the model in which each donor creates a single energy level $\varepsilon_0 < 0$, which may be occupied buy 0, 1 or 2 spinful noninteracting electrons. Each impurity brings exactly one mobile electron to the system. Find the number of conduction electrons as a function of temperature.

Let

$$m^* = m_e/10 \ , \ \varepsilon_0/k_B = -100\text{K} \ , \ n_d = 10^{14}\text{cm}^{-3}. \tag{7}$$

At what temperature will the majority of electrons move from the impurities to the conduction band?

Hint. Electrons on the impurity obey Fermi statistics. However, since the number of impurities is small compared to the total number of atoms, Fermi statistics never play any important role for the conduction electrons. **[2p]**