Algebra III (Algebraic Geometry II) WS15-16
Fachbereich Mathematik und Informatik
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Algebraic Geometry II

Exercise Sheet 5

Hand-in date: 10am, Monday 16th November.

Exercise 1.

a) Show directly that the additive group $\mathbb{G}_a$ is not geometrically reductive.

b) Show that any finite group of order not divisible by the characteristic of k is linearly reductive.

Exercise 2. Prove that the general linear group GL_n is reductive.

[Hint: for a contradiction, suppose we have a non-trivial smooth connected unipotent normal algebraic subgroup $U \subset \mathrm{GL}_n$, then there exists $g \in U(k) \subset \mathrm{GL}_n(k)$, which is unipotent and not equal to the identity. In particular, the Jordan normal form of g has a $r \times r$ Jordan block for $r > 1$. Then using normality of U, show that we can conjugate g to another element $g' \in U(k)$ such that the product gg' is not unipotent.]

Exercise 3. Let G be an affine algebraic group acting on a scheme X. If $\varphi : X \to Y$ is a good (resp. geometric) quotient, prove that for every open $U \subset Y$ the restriction

$$\varphi|_{\varphi^{-1}(U)} : \varphi^{-1}(U) \to U$$

is also a good (resp. geometric) quotient of G acting on $\varphi^{-1}(U)$.