Complex Analysis SS15
Fachbereich Mathematik und Informatik
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Complex analysis
Summer semester 2015

Exercise Sheet 8

Hand-in date: 12:00, Friday 5th June.

Exercise 1. Prove the Fundamental Theorem of Algebra using the minimum-modulus property.

Exercise 2.

a) Let $f : \overline{\mathbb{H}} \to \mathbb{C}$ be a function on $\overline{\mathbb{H}} := \{z \in \mathbb{C} : \operatorname{Im} z \geq 0\}$ which is bounded and analytic. If additionally f is real on the real axis, show that f is constant.

b) Let f be an entire function which takes real values on the real axis and imaginary values on the imaginary axis. Then show that f is an odd function i.e. $f(-z) = -f(z)$.

c) Let $S := \{z \in \overline{D_1(0)} : \operatorname{Im} z > 0\}$ and let $f : S \to \mathbb{C}$ be an analytic function, which is real on the boundary ∂S. Then prove that f has an analytic extension to the open upper half plane $\mathbb{H} := \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$.

Exercise 3. Classify the singularities of the following functions at the given points. For removable singularities z_0, calculate the value of the function as $z \to z_0$. For poles, calculate the order of the pole.

a)
$$\frac{1}{z^2 + z^4} \quad \text{at} \quad 0, \pm i.$$

b)
$$\frac{1}{1 - \exp z} \quad \text{at} \quad 0.$$

c)
$$\frac{1}{z - \sin z} \quad \text{at} \quad 0.$$

d)
$$\frac{\exp(1/z^2)}{z - 1} \quad \text{at} \quad 0, 1.$$