Complex Analysis SS15
Fachbereich Mathematik und Informatik
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Complex analysis
Summer semester 2015

Exercise Sheet 7

Hand-in date: 12:00, Friday 29th May.

Exercise 1. Let $D \subset \mathbb{C}$ be a domain and $\{f_n : D \to \mathbb{C}\}_{n\in\mathbb{N}}$ be a sequence of analytic functions which converges compactly to $f : D \to \mathbb{C}$. Prove that f is analytic and the sequence of kth derivatives $\{f_n^{(k)} : D \to \mathbb{C}\}_{n\in\mathbb{N}}$ converges compactly to $f^{(k)}$.
[Hint: to prove f is analytic, use Morera's Theorem.]

Exercise 2. Determine the radius of convergence of the Taylor series expansions of the following functions at the origin 0 and write down their Taylor series in an open ball around the origin:

a) $\frac{2z+1}{(z^2+1)(z+1)^2}$

b) $\exp(z)\cos(z)$

c) $\sin^2 z$

Exercise 3. Let $f : \overline{D_r(a)} \to \mathbb{C}$ be a continuous function on the closed ball which is analytic on the open ball $D_r(a)$. Prove the following statements:

a) The modulus $|f|$ attains its maximum on the boundary $\partial D_r(a)$. Furthermore, if f is non-constant, then for $z \in D_r(a)$,
$$|f(z)| < \max\{|f(w)| : w \in \partial D_r(a)\}$$
b) If f has no zeros in $D_r(a)$, then $|f|$ attains its minimum on $\partial D_r(a)$. Furthermore, if f is non-constant, then for $z \in D_r(a)$,
$$|f(z)| > \min\{|f(w)| : w \in \partial D_r(a)\}$$
c) If $|f|$ is constant on the boundary $\partial D_r(a)$, then show either f is constant or f has a zero.