2. Thermal Emission

2.1 Blackbody Radiation
   - Kirchhoff’s Law
   - Planck’s Function
   - Wien’s Displacement Law
   - Stefan-Boltzmann Law
   - Rayleigh-Jeans Approximation

2.2 Emissivity
   - Monochromatic Emissivity
   - Graybody Emissivity
   - Brightness Temperature
Consider an ‘isolated cavity’ and a hypothetical radiating body at temperature $T$.

An equilibrium will exist between the radiation emitted from the body and the radiation that body receives from the walls of the cavity.

The ‘equilibrium’ radiation inside the cavity is determined solely by the temperature of the body. This radiation is referred to as black-body radiation. Two black-bodies of the same temperature emit precisely the same amount of radiation.
Cavity radiation - experimental approximation to black-body radiation

Although the concept of blackbody radiation seems abstract there are a number of very practical reasons to devise ways of creating such radiation. One important reason is to create a source of radiation of a known amount that can be used to calibrate instruments. We can very closely approximate blackbody radiation by carefully constructing a cavity and observing the radiation within it.

Cavities are used both to create a source of blackbody radiation and also as a way of detecting all radiation incident through the cavity aperture.

Cavities are designed to be light traps - any incident radiation that emerges from the cavity experiences many reflections. If the reflection coefficient of the walls is low, then only a very tiny amount of the energy of incident radiation emerges - most comes from the radiation emitted by the walls of the cavity.
Other sources of radiation

Air mass 1.5 solar spectrum

Solar radiation at the ground

Quartz-halogen lamp
Kirchoff’s law

Suppose that \( B_\lambda(T) \) is the amount of blackbody radiation emitted from our hypothetical blackbody at wavelength \( \lambda \) and temperature \( T \). Then

\[
E_\lambda = a_\lambda B_\lambda(T)
\]

is the amount of radiation emitted from any given body of temperature \( T \). \( a_\lambda \) is the absorptivity or emissivity of the body. Its magnitude and wavelength dependence is solely determined by the properties of the body – such as the composition, and state (gaseous or condensed).

It is often reasonable to suppose that for some bodies \( a_\lambda = \text{constant} \) and these are referred to as ‘grey bodies’:

\[
a_\lambda = 1 \text{ for a black body}
\]

For some applications it is sometimes convenient to introduce the notion of a grey body emissivity defined as

\[
\varepsilon(T) = \frac{\int a_\lambda B_\lambda(T) d\lambda}{\int B_\lambda(T) d\lambda}
\]
Emissivity of the surfaces:

- In general, emissivity depends on the direction of emission, surface temperature, wavelength and some physical properties of the surface (e.g., the refractive index).
- In thermal IR ($\lambda > 4 \ \mu m$), nearly all surfaces are efficient emitters with the emissivity $> 0.8$ and their emissivity does not depend on the direction. Therefore, the intensity emitted from a unit surface at a given wavelength is $I_\lambda = \varepsilon_\lambda \ B_\lambda(T_s)$.

Table 3.1 Emissivity of some surfaces in the IR region from 10-to12 $\mu m$.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.993-0.998</td>
</tr>
<tr>
<td>Ice</td>
<td>0.98</td>
</tr>
<tr>
<td>Green grass</td>
<td>0.975-0.986</td>
</tr>
<tr>
<td>Sand</td>
<td>0.949-0.962</td>
</tr>
<tr>
<td>Snow</td>
<td>0.969-0.997</td>
</tr>
<tr>
<td>Granite</td>
<td>0.898</td>
</tr>
</tbody>
</table>
Empirical Radiation Laws

\[ B_\lambda = \frac{C_1}{\pi \lambda^5 \left( e^{\frac{C_2}{\lambda T}} - 1 \right)} \]

Wien Radiation Law (1893)

\[ B_\lambda = \frac{C_1}{\pi \lambda^5 (e^x - 1)} \]

\[ e^x \gg 1 \]

\[ B_\lambda \approx \frac{C_1}{\pi \lambda^5} \]

Applies at the shorter wavelengths

Rayleigh-Jeans Radiation Law (1894)

\[ B_\lambda = \frac{C_1}{\pi \lambda^5 (e^x - 1)} \]

\[ e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \]

\[ B_\lambda \approx \frac{C_1}{\pi \lambda^5 (1 + x - 1)} \]

\[ B_\lambda \approx \frac{C_1}{\pi \lambda^5 (x)} \]

\[ B_\lambda \approx \frac{C_1 T}{C_2 \pi \lambda^4} \]

Breaks down at small wavelengths - the UV catastrophe. This law only applies in the microwave - radiant energy and temperature are synonymous

Wien Displacement Law

\[ \lambda_{\max} \ T = 2898 \ \mu m \cdot K \]
Planck’s blackbody function (1900)

The nature of $B_\lambda(T)$ was one of the great findings of the latter part of the 19th century and lead to entirely new ways of thinking about energy and matter. Early experimental evidence pointed to two particular characteristics of $B_\lambda(T)$:

\[ B_\lambda = \frac{2\pi h c^2}{\pi \lambda^5 \left( e^{\frac{h c}{\lambda k T}} - 1 \right)} \]
\[ B_\lambda = \frac{C_1}{\pi \lambda^5 \left( e^{\frac{C_2}{\lambda T}} - 1 \right)} \]

\[ C_1 = 2\pi h c^2 = 3.7141832 \times 10^8 \text{ W} \cdot \text{um}^4 \cdot \text{m}^{-2} = 3.7141832 \times 10^4 \text{ W} \cdot \text{um}^4 \cdot \text{cm}^{-2} = 3.7141832 \times 10^{-4} \text{ W} \cdot \text{nm}^4 \cdot \text{m}^{-2} \]

\[ C_2 = h c / k = 14387.86 \text{ um} \cdot \text{K} \]
<table>
<thead>
<tr>
<th>Wavelength [µm]</th>
<th>Irradiance [Wm(^{-2})µm(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>December</td>
<td></td>
</tr>
<tr>
<td>Cahalan</td>
<td></td>
</tr>
<tr>
<td>June</td>
<td></td>
</tr>
<tr>
<td>June</td>
<td></td>
</tr>
<tr>
<td>December</td>
<td></td>
</tr>
</tbody>
</table>

Planck's blackbody function - Solar irradiance
Planck's blackbody function – Solar irradiance

Cahalan

June

December
Planck's blackbody function – Solar irradiance

Cahalan

June

December
Planck’s blackbody function - Solar and Earth irradiance

Cahalan

June

December
Planck's blackbody function – Solar and Earth irradiance

Cahalan

June

December

T=288

T=265
Sun temperature: $5780 \, \text{K}$

Sun – Earth distance – summer: 152.10 Mill. Kilometer
- winter: 147.09 Mill. Kilometer

Solar constant – summer: $1321.9 \, \text{Wm}^{-2}$
Solar constant – winter: $1413.4 \, \text{Wm}^{-2}$

Global mean solar irradiance – summer: $231.3 \, \text{Wm}^{-2}$ (albedo=0.3)
Global mean solar irradiance – summer: $247.4 \, \text{Wm}^{-2}$ (albedo=0.3)

Earth temperature – $T=265 \, \text{K}$: $239.3 \, \text{Wm}^{-2}$
Earth temperature – $T=288 \, \text{K}$: $343.5 \, \text{Wm}^{-2}$