$$f(x,y) = \begin{cases} (x^2+y^2)\cdot \sin\left(\frac{1}{\sqrt{x^2+y^2}}\right), & \text{falls } (x,y) \neq (0,0) \\ 0 & \text{, falls } (x,y) = (0,0) \end{cases}$$

1. Partielle Ableitungen berechnen
2. Totale Diff'barkeit nachweisen
3. zeigen, dass partielle Abl. nicht stetig sind

(1) $\partial_x f(0,0) = 0, \quad \partial_y f(0,0) = 0$

(2) $f: \mathbb{R}^{2\,(n)} \to \mathbb{R}^{m}, \quad (x_0,y_0) = (0,0)$

$$\underbrace{f\left(\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} h_x \\ h_y \end{pmatrix}\right)}_{\to \mathbb{R}^m} = \underbrace{f\left(\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}\right)}_{\to \mathbb{R}^m} + \underbrace{J(x_0,y_0)}_{\mathbb{R}^{m\times n}} \cdot \underbrace{\begin{pmatrix} h_x \\ h_y \end{pmatrix}}_{\mathbb{R}^n} + r\left(\begin{pmatrix} h_x \\ h_y \end{pmatrix}\right)$$

mit $\lim\limits_{\substack{h\to 0 \\ h=\binom{h_x}{h_y}}} \frac{r\left(\binom{h_x}{h_y}\right)}{\sqrt{h_x^2+h_y^2}} = 0$

$$r\begin{pmatrix} h_x \\ h_y \end{pmatrix} = f\left(\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} h_x \\ h_y \end{pmatrix}\right) - f\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \underbrace{J(x_0,y_0)}_{(\partial_x f(x_0,y_0) \quad \partial_y f(x_0,y_0))} \cdot \begin{pmatrix} h_x \\ h_y \end{pmatrix}$$

$x_0 = 0, \; y_0 = 0$

$$= (h_x^2+h_y^2)\cdot \sin\left(\frac{1}{\sqrt{h_x^2+h_y^2}}\right)$$

$$\frac{\left\| r\binom{h_x}{h_y} \right\|}{\sqrt{h_x^2+h_y^2}} = \frac{h_x^2+h_y^2 \cdot \left|\sin\left(\frac{1}{\sqrt{h_x^2+h_y^2}}\right)\right|}{\sqrt{h_x^2+h_y^2}} = \underbrace{\sqrt{h_x^2+h_y^2}}_{\to 0} \cdot \underbrace{|\sin(\dots)|}_{\text{beschr.}}$$

$$\to 0$$