$$p : \mathbb{R} \to \mathbb{R}$$

$$x \longmapsto a_0 \cdot x^0 + a_1 \cdot x^1 + \ldots + a_d \cdot x^d$$

$$= \sum_{k=0}^{d} a_k \cdot x^k$$

$$p(x) = a_0 \cdot \underbrace{x^0}_{=1} + a_1 \cdot x^1 + \ldots + a_d x^d$$

<u>Beispiel:</u> $d=2$

$$p(x) = a_0 + a_1 x + a_2 x^2 \quad \Big| \; a_1, \ldots, a_d \in \mathbb{R}$$

$a_0 = 2, \quad a_1 = -1.5, \quad a_2 = 1$

$\hookrightarrow \quad p(x) = 2 + (-1{,}5)x + \underline{\underline{x^{2}}}$

<u>Der</u> Grad dieses Polynoms ist 2.

$$\mathbb{R}[x]_3 \ni p(x) = \underline{a_0} + \underline{a_1} x + \underline{a_2} x^2 + \underline{a_3} x^3$$

$$\ni q(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3$$

$$(p \underset{\uparrow}{+} q)(x) := p(x) + q(x)$$

Vektorraum-addition

$$= (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3 \in \mathbb{R}[x]_3$$

$\lambda \in \mathbb{R}$

$$(\lambda \cdot p)(x) := \lambda \cdot p(x)$$

Skalarmultiplikation

$$= \lambda \cdot (a_0 + a_1 x + a_2 x^2 + a_3 x^3)$$

$$= (\underline{\lambda \cdot a_0}) + (\underline{\lambda a_1})x + (\underline{\lambda a_2})x^2 + (\underline{\lambda a_3})x^3 \in \mathbb{R}[x]_3$$

Beziehung zwischen $\mathbb{R}[x]_3$ und $\mathbb{R}^4$?!

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$\updownarrow$$

$$\tilde{p} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad \text{ebenso:} \quad \tilde{q} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$(p+q)(x) = \ldots$$

$$\tilde{p} + \tilde{q} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_0 + b_0 \\ a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$$

$$(\lambda p)(x) = \ldots$$

$$\lambda \tilde{p} = \lambda \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \lambda a_0 \\ \lambda a_1 \\ \lambda a_2 \\ \lambda a_3 \end{pmatrix}$$

$$E_k = (1-x)^k$$

$$E_0 = (1-x)^0 = 1 \qquad \tilde{E}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E_1 = 1-x \qquad \tilde{E}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$E_2 = (1-x)^2 = 1 - 2x + x^2 \qquad \tilde{E}_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

$$E_3 = (1-x)^3 = 1 - 3x + 3x^2 - x^3 \qquad \tilde{E}_3 = \begin{pmatrix} 1 \\ -3 \\ 3 \\ -1 \end{pmatrix}$$

$$P = x^3 + x^2 - 2x + 1$$

$$\tilde{P} = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 1 \end{pmatrix}$$

$$\tilde{P} = \lambda_0 \cdot \tilde{E}_0 + \lambda_1 \cdot \tilde{E}_1 + \lambda_2 \cdot \tilde{E}_2 + \lambda_3 \cdot \tilde{E}_3$$

$$\Leftrightarrow \begin{pmatrix} 1 \\ -2 \\ 1 \\ 1 \end{pmatrix} = \lambda_0 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ -3 \\ 3 \\ -1 \end{pmatrix}$$

$$\Leftrightarrow \begin{aligned} 1 &= \lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 \\ -2 &= -\lambda_1 - 2\lambda_2 - 3\lambda_3 \\ 1 &= \lambda_2 + 3\lambda_3 \\ 1 &= -\lambda_3 \end{aligned}$$

$$P(x) = \lambda_0 \cdot E_0(x) + \lambda_1 \cdot E_1(x) + \lambda_2 E_2(x) + \lambda_3 \cdot E_3(x)$$