Generic deformations of matroid ideals

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Theorem (-,Kahle, Varbaro '12)
If Δ is the (d–1)*-skeleton of a d-dim, CM complex, then* $\beta_p(S/I_\Delta) = \beta_p(S/\text{gin}(I_\Delta))$.

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Theorem (-,Kahle, Varbaro '12)
If Δ is the (d–1)-skeleton of a d-dim, CM complex, then $\beta_p(S/I_\Delta) = \beta_p(S/\text{gin}(I_\Delta))$.

In general: $\beta_{i,j}(S/I) \leq \beta_{i,j}(S/\text{gin}(I))$.

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Conjecture (Stanley '77) *The h-vector of a matroid complex is a pure O-sequence.*

A pure O-sequence is the Hilbert function of some *artinian*, *monomial*, *level* algebra.

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Artinian reduction	$S/(I_\Delta + (\ell_i))$	●	●	●
WANT: weakgin(I)	$S/(\text{weakgin}(I_\Delta) + (x_i - x_j))$			
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A graded, CM[1] algebra S/I is level if: $0 \longrightarrow S(-a)^{\beta_p} \longrightarrow \cdots \longrightarrow F_0 \longrightarrow S/I \longrightarrow 0$

Theorem (-,Kahle, Varbaro '12)
If Δ is the (d–1)-skeleton of a d-dim, CM complex, then $\beta_p(S/I_\Delta) = \beta_p(S/\text{gin}(I_\Delta))$.

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Thank you for your attention!