Honest Certification and the Threat of Capture

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Abstract

This paper studies problems of capture in certification markets. It derives conditions under which reputation enables certifiers to resist capture. Moreover, it identifies a general principle of reputation models that favors concentration. This explains certifiers as efficient market institutions that sell reputation as a service to other firms. The analysis yields the following insights: 1) A demand for external certification exists, despite being costly and susceptibility to capture. 2) Low discount factors require a price of certification that exceeds the static monopoly price. 3) Price competition tends to a monopolization of certification markets. 4) Honest certification exhibits economies of scale and constitutes a natural monopoly.

Keywords: certification, collusion, bribery, reputation, natural monopoly
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1 Introduction

Since the seminal article of Akerlof (1970) economists recognize that asymmetric information has important effects on the allocation and distribution of resources. This gives market participants incentives to undertake costly actions to signal their private information (e.g. Spence 1973), invest in reputation (e.g. Klein and Leffler 1981), or issue warranties. In addition, asymmetric information may lead to a demand for certifying intermediaries who try to reduce asymmetries by inspecting a party’s private information and reveal findings publicly (e.g. Biglaiser 1993 and Lizzeri 1999). Examples of certifiers are laboratories that test consumer products, auditors who validate the accounts of firms, ISO registrars who certify quality standards of production processes, internet search engines that rank Web sites, and schools that certify the ability of students.

This paper focuses on the role of certifiers in reducing the amount of asymmetric information in the presence of threats of capture. Although commercial markets involving certification seem to function relatively well, there exists much anecdotal evidence that problems of capture are nevertheless a major concern. The most prominent recent example is probably the Andersen-Enron accounting scandal, where Andersen, as Enron’s accountant, falsely certified Enron’s accounts as accurate. Yet, there exist many other examples of capture. To name only three other cases that occurred in the first 6 months of 2002: In Januar 2002 the German government had to recall the meat of 26,000 cattle after it was uncovered that a certifying laboratory had performed 40,000 dodgy BSE-tests. In March 2002 Sony Pictures was fined $325,000 for certifying its own films by inventing fake reviews that it attributed to an actual newspaper. In June 2002 the US Federal Trade Commission stated that search engines that take money from Web sites in exchange for prominent placement should make that practice clearer to Web users.

Clearly, if market participants become aware and expect that certifiers are fraudulent, certification, even when technically possible, becomes worthless. Hence, the trustworthiness of certification is a crucial determinant of its success. Yet, concerns about capture are natural, because successful certifi-
cation affects the terms of trade and therefore gives producers incentives to influence it. It seems that in response to these concerns certification services are sometimes provided by non-profit organizations. Yet, still a large part of the certification market remains in the hands of commercial firms. The large public outcry about cases like Andersen-Enron only underscores the fact that, despite threats of capture, these markets function relatively well in general. In order to understand the functioning and possible failure of such markets, this paper studies the problems of capture in certification.

More specifically, we investigate the role of reputation as a remedy to capture and derive conditions under which it makes honest certification feasible. These conditions have the following implications for the functioning and structure of certification markets:

1. For low discount factors, honest certification requires prices that exceed the static monopoly price.

2. Honest certification exhibits economies of scale.

3. Price competition tends to a monopolization of markets.

4. A demand for costly external certification exists even if a producer has the ability to develop a reputation himself.

We discuss these results in turn. First, from the literature on reputation (e.g. Klein and Leffler 1981, Shapiro 1983), it is well known that reputation requires prices to exceed marginal costs. We show that this also holds for the current model, but in an extreme way: For low discount factors, even the static monopoly price is too low to sustain a reputation for honesty. This result is new and, at first sight, surprising, because the monopoly price yields the certifier the highest per period payoff. Hence, it maximizes the certifier's gain from honesty and one may therefore expect that it also minimizes the overall threat of capture. We will, however, show that the price of certification also affects the potential gain from capture and that, at the monopoly

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1For example, the first public statement of the German ministry of health after uncovering the certification scandal concerning BSE-tests of beef was to ban "profit pursuing" firms from certifying beef products.
price, this gain is declining in price. As a consequence, the overall threat of capture is minimized at some higher price.\textsuperscript{2}

To explain the other three results we first identify a simple, but general feature of reputation models that has not been much emphasized in the literature. Namely that in models of reputation there exist gains from concentration. This simple feature has important economic implications. Ultimately, it enables specialized certifiers to sell the good “reputation” as a service to other firms. That is, the principle explains external certifiers as efficient market institutions that provide reputation. More specifically, it is responsible for the result 2, 3, and 4.

As is well understood, reputation models thrive on a trade-off between the short run gain from cheating and the long run gain for honesty. That is, if the long run gain rises, the trade-off changes in favor of honesty. Hence, when the certifier expects a larger future demand for honest certification, capture becomes a less attractive option. In this sense, we obtain our second result that honest certification exhibits increasing returns to scale; honesty is sustainable at lower prices, i.e. less costly, when the number of (future) certification jobs is high. Hence, when a certifier serves the entire demand himself rather than sharing it with competitors, he can perform honest certification at a lower price. In a monopoly, therefore, an honest certifier is able to charge lower prices than in an oligopoly. This leads to our third result that price competition tends to a monopolization of the market. Our analysis, consequently, helps to explain the empirical observation that certification markets are highly concentrated.

Effectively, we show that honest certification is easier to sustain when certification is concentrated at one party. Although this is a general principle of reputation models, it has for us the important implication that it may explain the very existence of external, specialized certifiers: For low discount factors a reputation for honesty is only sustainable when there is one institution — the external certifier — who provides this reputation rather than

\textsuperscript{2}The underlying mechanism leading to prices that exceed monopoly prices is therefore fundamentally different from that in models of signalling (e.g. Bagwell and Riordan 1991).
many independent ones — the actual producers of goods.\(^3\) This explains our final result and solves the more fundamental, institutional question why producers demand external certification rather than build up their own reputation. Ultimately and much in the spirit of Adam Smith, we demonstrate gains from specialization in reputation building. However, rather than postulating these gains we derive them more endogenously. Solely due to these gains, we may explain specialized certifiers as effective sellers of reputation to other firms and, moreover, provide an economic justification for certifiers as an efficient market institution.

There exists a growing literature which analyzes certifiers as market institutions that reduce the amount of asymmetric information. Even though the literature regards capture as a severe threat to effective certification, the papers in this field do not address the issue directly. For instance, Lizzeri (1999) and Albano and Lizzeri (2001) focus on the amount of information revelation by certifiers. By assuming that the certifier announces and precommits to a certain revelation profile, they circumvent the problems of dishonest certification and reputation does not play a role. Biglaiser (1993) analyzes middlemen in matching markets to resolve problems of asymmetric information. Middlemen act as certifiers who, after a sunk investment, can observe the products’ quality on the basis of which they may offer a non-enforceable warranty. Using an informal reputation argument which induces middlemen to honor their warranties, the paper obtains an efficiency effect of middlemen, but does not study problems of capture. In contrast, Biglaiser and Friedman (1994) analyze collusion between middlemen and producers, but since middlemen cannot observe the product’s quality, they are not certifiers.

The remainder of the paper is as follows. The next section introduces the framework which we use to study the problem of capture. Section 2 analyzes the demand for certification for some given price of certification, when the certifier behaves honestly. Section 3 introduces the problem of capture and analyzes the behavior of a monopolistic certifier who is susceptible to capture. In Section 4 we study the effect of price competition and Section

\(^3\)Hence, the mechanism that drives this result is very different from the one in Kofman and Lawarrée (1993), who argue that external supervision is less susceptible to capture than internal supervision.
5 considers the effect of innovation. Section 6 focuses on the demand for external certification and identifies the gains of specialization in reputation building. Section 7 discusses the result and concludes. All formal proofs are collected in the appendix.

2 The Setup

Consider a market in which in each period $\tau = 1, 2, \ldots, \infty$ a different monopolistic producer enters with a single unit of some quality $q_\tau \in [0, 1]$. Quality is stochastic and drawn from a uniform distribution that is i.i.d. of the qualities in previous periods. The good’s quality represents the reservation price of consumers. Only the producer observes the quality. They are short-lived and leave the market after selling their good in a second price auction. Consumers observe the product’s quality after consumption. Since only the producer knows the quality of his good before purchase, the market exhibits informational asymmetries. Production costs are zero.

Without any further economic institutions, a producer cannot persuade consumers of the quality of his good. Since Akerlof (1970) it has become standard to compute the equilibrium outcome. Consumers have a belief $q_\tau^e$ about the offered quality which, in equilibrium, coincides with the actual expected quality $E\{q_\tau\}$. Hence, in the second price auction consumers bid the expected market quality $E\{q_\tau\}$ and the good is sold at a price $p = E\{q_\tau\}$. The argument leads to the following result.

**Lemma 1** Without certification the market outcome is $p_\tau = q_\tau^e = E\{q_\tau\} = 1/2$.

Due to asymmetric information all producers are pooled and consumers are only willing to pay a uniform price reflecting the average quality in the market. The price is therefore independent of actual quality and producers

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4The second price auction results in a standard monopoly price while circumventing numerous complications associated with letting the informed party take a publicly-observed action that may be interpreted as a signal.

5We focus on certification as the only way to reduce informational asymmetries and abstract from all other remedies such as signalling and warranties.
with a high quality sell their goods at a relatively low, average price. These producers would gain if they could prove their quality through certification.\(^6\) Hence, there exists a demand for certification.\(^7\)

### 3 Honest Certification

Now assume that producers may certify their product quality at some price \(p\). Suppose that certification is performed by a monopolistic certifier who can, at a cost \(c \geq 0\), determine the good’s quality perfectly and announce it publicly. This section derives the equilibrium outcome when the certifier certifies honestly and cannot be captured. Concerns of capture are analyzed in Section 4.

Consider a producer \(q_r\) who has to decide whether to certify his product at a price \(p\). If he certifies, his product will be sold at a price reflecting the true quality. Certification therefore leads to a profit of

\[
\pi^c(q_r) = q_r - p.
\]

If producer \(q_r\) decides to sell his good uncertified, he obtains some price \(q^e_r\), where \(q^e_r\) represents the consumer’s belief of the average quality of non-certified goods. This selling strategy leads therefore to a profit

\[
\pi^n = q^e_r,
\]

which is independent of the product’s actual quality \(q_r\).

It follows that the producer certifies only if\(^8\)

\[
\pi^c(q_r) \geq \pi^n. \tag{1}
\]

\(^6\)Hence, we focus on a distortion that is purely redistributive and not allocative. This enables us to address the positive questions of certification in a clear, tractable way. The final section discusses possible extensions that would enable a study of normative issues.

\(^7\)Section 7 establishes a demand for certification also when producers are long-lived and could establish a reputation themselves with a more efficient, i.e. costless, technology.

\(^8\)We assume that an indifferent producer certifies. Due to the uniform distribution this assumption is inconsequential.
Since the difference in the producer’s profit \( \pi^c(q_r) - \pi^n \) is monotonically increasing in \( q_r \), the market with certification is a partition equilibrium. That is, at most one producer \( \bar{q}_r \) is indifferent concerning certification. All producers \( q_r > \bar{q}_r \) have a strict preference for certification, while all producers \( q_r < \bar{q}_r \) do not certify.  

For the indifferent producer \( \bar{q}_r \) it holds \( \pi^c(\bar{q}_r) = \pi^n \) and it follows 
\[
\bar{q}_r(p) = q^e_r + p.
\]

Hence, all producers with a quality of at least \( \bar{q}_r(p) \) certify and, given \( \bar{q}_r(p) \in [0, 1] \), (expected) demand for certification is \( 1 - \bar{q}_r(p) \). Since, on the other hand, exactly those producers with a quality below \( \bar{q}_r(p) \) do not certify, the consumers’ belief \( q^e_r \) concerning the average quality of a non-certified product is, in equilibrium, \( \bar{q}_r(p)/2 \). It follows that the indifferent producer equals \( \bar{q}_r(p) = 2p \). Demand for certification is therefore 
\[
D^h(p) = 1 - 2p.
\]

The static monopoly price may be calculated from the certifier’s profit \( \Pi^h(p) = D^h(p)(p - c) \) as 
\[
p^m = (1 + 2c)/4 \text{ if } c \leq 1/2.
\]  
(2) Whenever the cost \( c \) exceeds \( 1/2 \), the certifier is unable to make a profit. In this case, profitable certification would require \( p > c > 1/2 \), but according to the market outcome of Lemma 1, even a producer with the highest quality 1 is unwilling to pay more than \( 1/2 \).

The certifier obtains a maximum overall profit, if he charges \( p^m \) in every period. This yields a discounted profit of 
\[
\sum_{t=0}^{\infty} \delta^t D^c(p^m)(p^m - c) = \frac{(1 - 2c)^2}{8(1 - \delta)},
\]
where \( \delta < 1 \) represents the discount factor.

## 4 Capture

The previous section assumed that the certifier reports his finding honestly. Yet, there exists a clear pressure from low quality producers to have their
product certified at a higher level. This section addresses this problem by introducing the possibility of capture.

To model the possibility of capture we use the collusion framework as initiated by Tirole (1986) and assume that the certifier and producer can side-contract and exchange transfers. Consumers cannot observe these side-contracts, but are fully aware of their possibility. The framework allows for an imperfect transfer technology, in that a bribe $b$ of the producer is only worth $\lambda b$ to the certifier, where $\lambda \leq 1$. The parameter $\lambda$ offers a convenient way to parameterize the potential threat of capture. E.g., the Sony Picture case, where the producer also acted as certifier, illustrates the extreme case $\lambda = 1$. In the Andersen-Enron case there were no direct monetary bribes. Instead, bribery was of a more inefficient, indirect nature so that $\lambda < 1$.

The possibility of capture is introduced as follows: After a producer $q_T$ enters, the certifier, without observing $q_T$, may make an offer $(b, q^b)$ to the producer. The offer describes the terms at which the certifier is willing to become captured, where $b$ represents the required financial transfer and $q^b$ the offered level of certification. If the producer accepts, he pays the bribe $b$, which has a value $\lambda b$ to the certifier, and his product is certified at quality level $q^b$. If the producer rejects the offer, he may still ask for an honest certification at price $p$. That is, a producer may insist on honest certification simply by rejecting any capture offer $(b, q^b)$ and, subsequently, paying the fee $p$. In this case, the certifier cannot manipulate the certification outcome. We motivate this assumption by following Kofman and Lawarée (1993) and assume that the certifier is unable to forge certification without the help of the producer.

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10 Hence, an explanation why Sony Pictures attributed its self-certifying efforts to an independent newspaper is that it tried to convince its consumers that $\lambda$ was low. Interestingly, Sony Pictures was not fined for its self-certifying activities perse, but for its claim that this “certification” was done by an independent newspaper.

11 Allegedly, Andersen obtained some lucrative complementary deals in exchange for its favorable certification services. A practise which the Sarbanes–Oxley Act of 2002 tries to prevent. We comment on this Act in the last section.
Within this framework the possibility of capture threatens honest certification for two reasons. First, producers with low qualities would like to capture the certifier to obtain a higher certification. Second, when captured the certifier saves the cost $c$. Hence, by allowing collusion before the certifier expends $c$ and observes actual quality, we may analyze these two threats simultaneously.\textsuperscript{12} The parameter $\lambda$ thereby measures the relative importance of the two. For $\lambda = 0$ only the cost saving argument threatens honest certification, whereas for $\lambda > 0$ also the incentives of the producer matter.

The main aim of this paper is to study how a reputation for honesty may prevent capture. Hence, we will investigate consumers who stop trusting a certifier once they detect a false testimony about a product’s quality. A certifier who anticipates this behavior may be prevented from issuing forged certification reports, because he knows that he will lose the trust and thereby the potential demand for certification in subsequent periods.

As shown in the previous section producers only certify in an honest equilibrium when their quality lies in the interval $[2p, 1]$. Hence, as soon as the certifier reports some quality outside this interval, it is evident to consumers that play diverged from the honest equilibrium. Consumers interpret such deviations as a sign that the certifier is dishonest and, subsequently, believe that the producer’s quality is zero.\textsuperscript{13}

To make the behavior of consumers more precise, let $h_\tau = (n_\tau^c, q_\tau^c, q_\tau)$ denote the certification outcome in period $\tau$, where $n_\tau^c \in \{0, 1\}$ indicates whether certification in period $\tau$ took place, $q_\tau^c$ represents the certifier’s claimed quality, and $q_\tau$ the actual quality observed after consumption. If certification in period $\tau$ did not take place, it holds $n_\tau^c = 0$ and $q_\tau^c = 0$. Now let $H_t = (h_1, \ldots, h_{t-1})$ summarize the history of certification outcomes at

\textsuperscript{12}The examples in the introduction show that both concerns are important. The Enron-Andersen and the Sony case illustrate capture, whereas the German BSE testing scandal was mainly attributed to the laboratory’s aim to cut costs even though producers were aware of the sloppy testing procedures.

\textsuperscript{13}For convenience, we resort in this paper to extremely pessimistic out-of-equilibrium beliefs. We emphasize however that such extreme beliefs are not crucial. They simply guarantee that a dishonest certifier receives zero profits after he has been exposed. We may achieve this also with milder out-of-equilibrium beliefs.
the beginning of period \( t \). Finally, let \( q^c_t(q^c_t, n^c_t, H_t) \) represent the consumers’ belief in period \( t \), when the consumers are faced with a certified quality \( q^c_t \) and have observed the certification history \( H_t \). The consumers’ behavior may then be captured by the following assumption about beliefs.

**Assumption 1** For the consumers’ belief \( q^c_t(q^c_t, n^c_t, H_t) \) it holds \( q^c_t(q^c_t, 1, H_t) = q^c_t \) whenever \( q^c_t \in [2p, 1] \) and \( \{ \tau < t | n^c_{\tau} = 1 \land q^c_{\tau} \neq q^c_t \} = \emptyset \). Moreover, \( q^c_t(q^c_t, 1, H_t) = 0 \) whenever \( \{ \tau < t | n^c_{\tau} = 1 \land q^c_{\tau} \neq q^c_t \} \neq \emptyset \) or \( q^c_t \notin [2p, 1] \).

The assumption states that consumers believe the certifier if he announces a quality in the interval \([2p, 1]\) and has not cheated in previous periods. Whenever the certifier did cheat in some previous period or announces some quality outside the interval, consumers do not trust the certifier’s claim and believe that the quality is in fact zero. In the following we derive conditions under which equilibria exist that satisfy Assumption 1. Since in equilibrium beliefs are confirmed, this implies that such equilibria are honest, in the sense that capture takes place with probability zero.

In order to derive an equilibrium in which certification is always honest, we proceed in two steps. First we analyze the potential threat of capture by studying the behavior of a producer when faced with a bribing offer \((b, q^b)\). In a second step, we derive the conditions under which the certifier will not make any offer \((b, q^b)\) that is acceptable to some producer \( q \in [0, 1] \). In this case, capture occurs with probability zero and Assumption 1 is consistent with the behavior of the certifier. Note that since the consumers trust a certification level of at most 1, a bribing offer with \( q^b = 1 \) poses the largest threat of capture. Hence, in the following we focus on such offers and simply talk of a bribing offer \( b \) rather than \((b, 1)\).

Suppose the certifier makes an offer \( b \) to some producer \( q^c_t \). If the producer accepts the offer, he receives a net profit \( 1 - b \). His profit from rejecting the offer depends on his quality \( q^c_t \) and the price of certification \( p \). If the producer’s quality exceeds \( 2p \), he would, according to the previous section, certify and receive a profit \( q^c_t - p \). Consequently, in an honest equilibrium a producer \( q^c_t \geq 2p \) rejects the offer only if \( 1 - b \leq q^c_t - p \). On the other hand, a producer with quality \( q^c_t < 2p \) does not certify in an honest equilibrium.
and receives, according to Section 3, a profit $p$. Hence, he rejects the offer if $1 - b \leq p$. Since a producer’s quality is uniformly distributed over $[0, 1]$, it follows that, in an honest equilibrium, the expected acceptance probability of a capture offer $b$ is

$$\alpha(b|p) = \begin{cases} 1 & \text{if } b < p \\ 1 + p - b & \text{if } b \in [p, 1 - p) \\ 0 & \text{if } b \geq 1 - p. \end{cases}$$

Figure 1 illustrates the acceptance probability graphically. Using this probability one may calculate the certifier’s expected payoff $V(b|p)$ from an offer $b$. For $b < p$ all producers accept the offer and the certifier obtains a profit $V(b|p) = \lambda b$. For $b \in [p, 1 - p]$ only producers $q_r < 1 + p - b$ accept, while producers $q_r \geq 1 + p - b$ reject and apply for honest certification. Hence, the certifier’s profit is $V(b|p) = (1-p-b)\lambda b + (b-p)(p-c+\delta V^h(p))$. Whenever $b \geq 1 - p$ all producers reject and the certifier obtains $V(b|p) = V^h(p) = (1-2p)(p-c)/(1-\delta)$.

The payoff $V(b|p)$ represents the certifier’s expected payoff from the offer $b$. If it exceeds the certifier’s payoff from honest certification, $V^h(p)$, the certifier is better off becoming captured with the associated probability $\alpha(b|p)$. Consequently, we may interpret $V(b|p)$ as the threat of the offer $b$ to honest certification. We say that certification at a price $p$ is capture proof if and only if

$$V^h(p) \geq V(b|p) \quad (3)$$
for all $b$. That is, in an honest equilibrium condition (3) must hold. Moreover, only in this case, Assumption 1 is consistent with equilibrium play such that we obtain indeed a Perfect Bayesian Equilibrium in which reputation prevents capture. The following proposition derives a condition under which such capture proof equilibria exist.

**Proposition 1** An equilibrium satisfying Assumption 1 is capture proof. It exists if and only if

$$
\delta \geq \delta^c(p) \equiv \frac{\lambda(1 - p)}{\lambda(1 - p) + (1 - 2p)(p - c)}.
$$

The proposition shows that the discount factor plays a crucial role for the existence of honest, i.e., capture proof, equilibria. As is well-known from the standard literature on repeated games, it determines the relative weights of the short run gain, the bribe $b$, and the long run loss of capture, relinquishing future orders for certification. Since the price of certification $p$ determines the certifier’s profit of future orders, the critical discount factor, $\delta^c(p)$, itself depends on the price of certification $p$. Figure 2 plots the typical shape of the curve $\delta^c(p)$. The shaded area illustrates the combinations of $(p, \delta)$ for which capture proofness is sustainable. As formally proved in the appendix the curve $\delta^c(p)$ is convex and obtains a minimum. These properties of $\delta^c(p)$ yield the following insight.

**Proposition 2** For any discount factor $\delta \geq \delta^*$ there exists an interval of prices $[p_l(\delta), p_h(\delta)]$ that sustain truthful certification, where

$$
\delta^* \equiv \frac{\lambda}{3 - 2\sqrt{2 - 2c - 2c + \lambda}}
$$

and

$$
p_l(\delta) \equiv \min_p \{p | \delta^c(p) = \delta\} \text{ and } p_h(\delta) \equiv \max_p \{p | \delta^c(p) = \delta\}.
$$

Hence, for a given discount factor $\delta > \delta^*$ there exist multiple prices that sustain truthful certification. The most preferable price from the perspective
of the certifier is the monopoly price $p^m$ as, under honest certification, this price yields the certifier the highest payoff. Yet, as Figure 2 indicates, at relatively low discount factors honest certification may require a price that exceeds the monopoly price $p^m$.

**Proposition 3** It holds $\delta^* < \delta^c(p^m)$. Moreover, for all $\delta \in [\delta^*, \delta^c(p^m))$ only prices that exceed the static monopoly price $p^m$ sustain honest certification.

At first sight the result is counter intuitive. The static monopoly price yields the certifier the highest per period payoff and, thereby, maximizes the long run penalty from becoming captured, i.e., losing future monopoly profits. This suggests that a monopoly price minimizes the overall threat of capture. Yet, the argument neglects that also the short run gain from capture depends on the price $p$. Indeed, at the monopoly price $p^m$ there is no first order effect of a price change on the certifier’s profit. Hence, the question whether the critical discount factor $\delta^c$ increases or decreases at $p^m$ depends only on the effect of a price change on the short run gain. Figure 1 reveals that a raise in $p$ has an ambiguous effect. On the one hand, a higher $p$ reduces the maximum bribe $b = 1 - p$ that non-certifying producers are willing to pay. On the other hand, a higher $p$ raises the minimum bribe $b = p$ at which all producers accept. The following lemma shows that the
maximum threat of capture comes from an offer \( b = 1 - p \) such that the relevant effect is the former one.

**Lemma 2** In a capture proof equilibrium the threat of capture, \( V^c(b|p) \), is maximized for \( b = 1 - p \). At the monopoly price \( p^m \) the maximum threat of capture, \( V^c(1 - p|p) \), is decreasing in \( p \).

From Lemma 2 it follows that a marginal increase in the monopoly price \( p^m \) reduces the overall threat of capture and therefore allows a reduction of the critical discount factor. Consequently, \( \delta^c(p) \) is decreasing at \( p^m \) and obtains its minimum \( \delta^* \) at a price that exceeds \( p^m \). This explains the result of Proposition 3. It also shows that the principle that underlies this result differs from that found in the literature on signalling (e.g. Bagwell and Riordan 1991), where a credible signalling of high quality may require prices exceeding the monopoly price.

Until now we investigated the existence of capture proof equilibria for some exogenously given price of certification \( p \). In the remainder of this section we look at the optimal pricing behavior of a monopolistic certifier.\(^{14}\) In this case, the price of certification becomes an explicit strategic variable and consumers may interpret it as a signal about the certifier’s honesty. We must therefore extend \( q^c_t(q^c_t, n^c_t, H_t) \) to include \( p \) and write \( q^c_t(q^c_t, n^c_t, H_t, p) \). Since out-of-equilibrium belief regarding the price of certification are, in principle, arbitrary, a multiplicity of equilibria obtains. For instance, it is easy to sustain any price \( \bar{p} \in [p_l(\delta), p_h(\delta)] \) as an equilibrium price by specifying that consumers interpret all other prices as set by a dishonest certifier. Such out-of-equilibrium beliefs prevent the monopolist from charging any other price than \( \bar{p} \). To avoid such arbitrariness, we extend Assumption 1 to all prices that, in principle, can sustain honest certification.

**Assumption 2** For the consumers’ beliefs it holds \( q^c_t(q^c_t, 1, H_t, p) = q^c_t \) whenever \( \delta(p) \leq \delta, q^c_t \in [2p, 1] \), and \( \{\tau < t | n^c_\tau = 1 \land q^c_\tau \neq q_\tau\} = \emptyset \). Moreover, for \( \delta(p) \leq \delta \) it holds \( q^c_t(q^c_t, 1, H_t, p) = 0 \) whenever \( \{\tau < t | n^c_\tau = 1 \land q^c_\tau \neq q_\tau\} \neq \emptyset \) or \( q_\tau^c \not\in [2p, 1] \).

\(^{14}\)Formally, we extend the game by an initial stage, where the certifiers chooses the price \( p \). The equilibrium in Proposition 1 is then an equilibrium of the subgame given a price \( p \).
The principle underlying the belief formation is that consumers trust a certifier if they have no compelling reason to mistrust him. That is, consumers treat equally all prices that may sustain honest certification. Like in Assumption 1, they anticipate that, for these prices, a certifier has no incentive to become captured. The following proposition shows that the assumption yields a definite equilibrium outcome.

**Proposition 4** In any equilibrium satisfying Assumption 2 certification takes place if and only if \( \delta \geq \delta^* \). In any such equilibrium the monopolistic certifier sets \( \hat{p}^m = \max\{p^m, p_l(\delta)\} \). Whenever \( \delta < \delta^* \) honest certification is not sustainable and the monopolist’s profit is zero.

The proposition shows that the monopolist’s pricing behavior depends on the discount factor. First, for large discount factors the static monopoly price is able to sustain honest certification and, since this price gives the highest per period payoff, the monopolist has no incentive to deviate from it. In this case, the certifier charges higher prices than needed to sustain honest certification. Second, when the discount factor is small, consumers foresee that even high prices of certification cannot prevent the certifier from becoming captured. Consequently, they will not trust the certifier and the monopolist is unable to derive a profit from certification. Third, for intermediate values of the discount factor, the static monopoly price is unable to sustain honest certification. This forces the monopolist to charge a price that exceeds the static monopoly price. Interestingly, only in this latter case does the monopolist charge the lowest price at which certification is sustainable.

5 Price Competition

For two reasons we examine in this section the effect of price competition. First, the previous section showed that for relatively high discount rates, a monopolistic certifier charges prices that are higher than the minimum price at which honest certification is sustainable. This raises the question whether price competition from competing certifiers may lead to lower prices. Second and more importantly, the sustainability of honest certification depends crucially on the possibility of maintaining prices that exceed costs. Since
price competition tends to lower prices, it may undermine this possibility and thereby the viability of certification markets.

To address these questions we first extend the monopolistic model to allow for price competition in a natural way. Suppose there exist \( N > 1 \) certifiers who each, at the beginning of the game, commit to a price of certification \( \pi_j \).\(^{15}\) All certifiers face the same discount factor \( \delta \). They are equally efficient, i.e., may all certify a product at costs \( c \). After setting prices producers enter and exit sequentially. We assume that the producer’s choice whether and where to certify is observable by consumers and subsequent producers. We assume further that each producer uses at most one certifier.

Moreover, let \( n^c_\tau \in \{0, \ldots, N\} \) in the consumers’ information set \( H_\tau \) denote the certifier who performed the certification in period \( \tau \). Whenever certification did not take place, it holds \( n^c_\tau = 0 \) and \( q^c_\tau = 0 \). The consumers’ beliefs in period \( t \) may be written as \( q^c(q^c_t, n^c_t, H_t, p) \), where \( p = (p_1, \ldots, p_N) \) is the vector of prices set by the certifiers.

Before analyzing the market game with price competition between certifiers, we first establish a preliminary result concerning the relation between market structure and the price of certification \( p \).

**Lemma 3** Suppose \( \delta \geq \delta^* \). At a price \( p_t(\delta) \) honest certification is sustainable only if the entire demand for certification is satisfied by a unique certifier.

Effectively, the proposition shows that the lowest price at which honest certification is sustainable is only attainable in a monopoly. The reason for this is straightforward. Honest certification depends on the threat that a certifier loses enough future payoffs. Yet, if multiple certifiers are active, the expected number of future certification jobs for a single certifier is smaller, as total demand is shared with others. In order to compensate for the reduced number of jobs the benefits of a single job, i.e., its price, must be larger to prevent capture.

This intuition indicates that, even though certification itself is a technology with constant returns to scale, honest certification exhibits increasing

\(^{15}\)As we will discuss in Section 8 commitment is not a crucial assumption.
returns to scale. If future demand for certification is higher, then honest certification can be achieved at lower prices, i.e., from the perspective of producers, at lower costs. Note that we obtained this feature endogenously, implying that the increasing return to scale has an economic rather than a technological origin. It moreover suggests that certification markets display characteristics of a natural monopoly. Indeed, the main result of this section is that price competition leads to a market outcome in which only one certifier is active and charges the lowest price at which honest certification is sustainable.

As before, we have to address the out-of-equilibrium beliefs to obtain a definite equilibrium outcome. We therefore adapt Assumption 2 as follows.

**Assumption 3** If $p_k \in \min\{p_j|\delta(p_j) < \delta\}$, $q^c_k \in [2p_k, 1]$ and $\{\tau < t|n^c_\tau = k \wedge q^c_\tau \neq q_\tau\} = \emptyset$, then $q^c_t(q^c_t, k, H_t, p) = q^c_t$. Moreover, if $n^c_\tau > 0$ then $q^c_t(q^c_t, n^c_\tau, H_t, p) = 0$ whenever $\{\tau < t|n^c_\tau = n^c_\tau \wedge q^c_\tau \neq q_\tau\} \neq \emptyset$ or $q^c_t \not\in [2p_{n_\tau}, 1]$.

The assumption states that consumers trust certifiers who charge the lowest price $\delta(p_j) < \delta$. It is therefore weaker than a straightforward extension of Assumption 2 to all certifiers. The following proposition shows that the assumption is, nevertheless, strong enough to yield a definite equilibrium outcome.

**Proposition 5** An equilibrium in which Assumption 3 is satisfied exists. In any such equilibrium certification is performed honestly by a unique certifier at price $p_t(\delta)$.

The proposition shows that price competition leads to a monopolization of the market. The driving mechanism leading to the result is Lemma 3. Assumption 3 thereby only ensures that price competition “works” in that it drives down prices. As in the standard Bertrand competition model, it gives a certifier an incentive to undercut any price $p > p_t(\delta)$. Yet, a certifier cannot effectively undercut the price $p_t(\delta)$, because consumers anticipate that at such prices, the certifier will not stay honest. Hence, the only remaining candidate is the price $p_t(\delta)$, but according to Lemma 3 honest certification
at this price is only sustainable if a unique certifier performs all certification. This requires that in any equilibrium that satisfies Assumption 3 producers coordinate and single out one certifier.

Proposition 5 answers the two questions that we asked in the beginning of this section. First, potential price competition from competitors does indeed lead to lower prices. Second, price competition does not destroy the possibility to charge prices above marginal costs. The reason is that a certifier cannot undercut the price $p_l(\delta)$ effectively, because consumers anticipate that honest certification is not viable at such prices.

6 Stochastic Innovation

As an extension we investigate a market of certification with stochastic innovation. We first derive a preparatory result concerning price competition with different marginal costs. Consider a situation in which the cost of certification differs between certifiers with $c^1 < c^2 < \ldots < c^N$. Let $\delta^*(c)$ represent the minimum discount factor for which honest certification is sustainable by a monopolist with certification costs $c$. For $\delta \geq \delta^*(c^i)$ let $p^i_l = p_l(\delta, c^i)$ and $p^i_h = p_h(\delta, c^i)$ denote the respective minimum and maximum price that would sustain honest certification. Moreover, let $p^m(c)$ represent the static monopoly price for a cost $c$ as defined by (2).

With different marginal costs, certifiers differ in their attitude towards capture. In particular, the minimum price $p^i_l$ at which a certifier is able to sustain honest certification as a monopolist is increasing in the cost $c$ such that it holds $p^1_l < \ldots < p^N_l$. If we want to maintain the idea that consumers will trust only certifiers who have no incentive to behave dishonestly, we have to adapt Assumption 3.

Assumption 4 If $p_i \in \min_j \{p_j | p_j \in [p^j_l, p^j_h] \}$, $q^c_i \in [2p_i, 1]$ and $\{ \tau < t | n^c_\tau = i \wedge q^c_\tau \neq q^r_\tau \} = \emptyset$, then $q^c_i(p^c_i, i, H_t, p) = q^c_i$. Whenever $\{ \tau < t | n^c_\tau = i \wedge q^c_\tau \neq q^r_\tau \} \neq \emptyset$, $q^c_i \not\in [2p_i, 1]$, or $p_i \not\in [p^i_l, p^i_h]$, it holds $q^c_i(p^c_i, i, H_t, p) = 0$.

The assumption places an additional restriction on the beliefs concerning certifiers with relatively high costs. Such certifiers may charge a price ex-
ceeding $p_1^l$ that is still too small to sustain honest certification. For this case, Assumption 4 specifies that consumers anticipate capture and consequently believe that the certified quality is zero. This restriction was not needed in the previous section, since all costs were identical.

**Proposition 6** In any equilibrium that satisfies Assumption 4 certification takes place if and only if $\delta \geq \delta^*(c^1)$. In any such equilibrium certifier 1 performs all certification at a price of certification $p^*(c^1, c^2)$ where

$$p^*(c^1, c^2) = \begin{cases} 
  p_1^l & \text{if } p_1^l \geq p^m(c^1) \\
  p^m(c^1) & \text{if } p_1^l < p^m(c^1) < p_2^l \\
  p_2^l & \text{if } p_2^l \leq p^m(c^1).
\end{cases}$$

The proposition resembles the standard Bertrand outcome of competition between producers with homogenous goods but different marginal costs. A subtle difference, however, is that, contrary to the Bertrand case, certifier $i$ is unable to attract any demand if he sets his price below $p_1^l$. This prevents certifier 2 from successfully undercutting certifier 1 and enables certifier 1 to charge the price $p^*(c^1, c^2)$.

We introduce the possibility of stochastic innovation as follows. The certification game starts with two certifiers with different costs $c^0$ and $c^1$, where $c^1/c^0 = \gamma < 1$. After the two certifiers commit to prices $p^0$ and $p^1$, a producer $q_1 \in [0, 1]$ enters and the decisions concerning capture and certification are taken. After the producer $q_1$ leaves the market, a new certifier with costs $\gamma c^1$ enters with probability $\pi$. That is, if entry occurs, the cost of the most efficient technology drops to $\gamma c^1$ and the entrant commits to some price $p_2^l$.

The idea we investigate here is that the more efficient entrant may displace the incumbent through price competition such that the entrant takes on the role of incumbent in future periods. Yet, at the beginning of period 3 again a more efficient certifier with costs of certification $\gamma c^n$ may enter and set some price $p^3$. More precisely, we assume that entry of a more efficient certifier occurs with probability $\pi$ at the beginning of every period. In the following let $c_\tau$ denote the most efficient technology is period $\tau$. That is, $c_1 = c^1$ and...
$c_\tau = \gamma c_{\tau - 1}$ whenever entry occurs. In this case, let $p^\tau$ represent the price to which the entrant commits himself. If entry in period $\tau$ does not take place, $c_\tau = c_{\tau - 1}$.

Note that if the more efficient entrant does displace the incumbent, the incumbent’s profit from future certification jobs is zero. In this case, the situation is as if the effective discount rate were $\hat{\delta} \equiv (1 - \pi)\delta$ rather than $\delta$. Hence, Proposition 6 implies that in period $\tau$ only the most efficient certifier is active. This confirms that after entry the entrant does indeed displace the incumbent and, therefore, establishes the existence of such an equilibrium.

In the remainder of this section we investigate so-called non-dramatic innovations that do not give monopoly power to the most efficient certifier. Whether an innovation is non-dramatic depends on the magnitude of $\gamma$. In particular, the cost difference between certifier 0 and 1 is non-dramatic if $p_t(\hat{\delta}, c^1) < p^m(\gamma c^1)$, as in this case certifier 0 can profitably undercut the monopoly price of certifier 1. The following lemma shows that this condition guarantees that all subsequent innovations also lead to non-dramatic cost differences.

**Lemma 4** If $p_t(\hat{\delta}, c^2) < p^m(\gamma c^2)$ then $p_t(\hat{\delta}, c) < p^m(\gamma c)$ for all $c < c^2$.

For non-dramatic innovations we obtain the following result.

**Proposition 7** There exists an equilibrium in which in every period $\tau$ the most efficient certifier serves the demand of certification at a price $p_t(\hat{\delta}, c_{\tau - 1})$. The market price converges to $p_t(\hat{\delta}, 0)$, which is increasing in the rate of innovation $\pi$.

The proposition shows that certification markets with a higher degree of innovation require higher prices to sustain honest certification. The reason for this is straightforward. In a market with stochastic innovation the future rewards from honest certification are lower, because with some probability the certifier loses future rents to a new entrant. To keep honest certification nevertheless sustainable the price of certification must be larger.
7 A Demand for External Certification

Until now we assumed that the certifiers are long-lived, whereas producers were short-lived. Producers therefore could not build a reputation themselves and this asymmetry created a demand for external certification. The results in the previous sections, however, indicate that a demand for external certification may exist even when producers are also long-lived. The idea is that by pooling the certification jobs of different producers it is easier for an external certifier to build up a credible reputation than for each producer individually. This section investigates this issue more closely.

Let there be $m$ long-lived producers. Producers produce sequentially, in that a producer $a \in \{1, \ldots, m\}$ produces a single good in periods $a, m + a, 2m + a, \ldots$. The quality of a producer differs over the periods and is drawn i.i.d. from the uniform distribution over $[0, 1]$. As before, the producer observes the quality directly, whereas consumers only observe it after consumption. The good is sold in a second price auction and the discount factor is $\delta$.

In this extension a producer may try to build his own reputation by announcing his current quality before each auction. In line with our previous belief restrictions, let consumers believe the producer’s announcement if they have no evidence that the producer has cheated on previous ones. Otherwise, they believe that the quality is zero. These beliefs are only confirmed in equilibrium if the producer’s announcement is indeed truthful. Hence, a necessary condition for the existence of such an equilibrium is that no producer has an incentive to cheat and claim a false quality. Clearly, a producer with a current quality of zero has the strongest incentive to cheat and announce a quality of 1 in order to receive a current profit of 1. Instead,

\footnote{The sequential structure is not crucial, but allows us to apply our previous results directly to this extension.}

\footnote{Clearly, if the quality of a producer is drawn only once and remains fixed over the periods, reputation is not sustainable in equilibrium. A producer with a quality close to zero always has an incentive to mimic higher ones.}

\footnote{Again, these pessimistic beliefs represent the most favorable case for reputation building, but may be relaxed.}
when the producer remains honest, he receives an expected payoff of
\[ 0 + \sum_{t=1}^{\infty} (\delta^m)^t \frac{1}{2} = \frac{\delta^m}{2 - 2\delta^m}, \]
as his expected quality in each future period is \( \frac{1}{2} \). Hence, producers are able to build up their own reputation only if
\[ \delta \geq \delta^e \equiv \left( \frac{2 \sqrt{3}}{3} \right)^{1/m}. \]

Now consider that as an alternative to building his own reputation a producer may turn to an external certifier. Clearly, whenever \( \delta \geq \delta^e \), it will not be profitable to do so, because external certification requires an additional cost \( p \) without providing additional services. Hence, the question is whether an external certifier is able to deliver honest certification for lower discount factors than \( \delta^e \). This is the case, whenever the minimum discount factor that sustains honest certification \( \delta^* \) is lower than \( \delta^e \). In this case, a demand for external certification exists at a price \( p^* \), because a producer with a quality close to 1 gains from such certification.

**Proposition 8** There exists a demand for external certification for discount factors \( \delta \in [\delta^*, \delta^e] \), whenever
\[ \frac{\lambda}{3 - 2\sqrt{2} - 2c - 2c + \lambda} < \left( \frac{2}{3} \right)^{1/m}. \]  \( (4) \)

The proposition shows that there is a potential demand for external certification if the number of producers, \( m \), is large and the parameters \( \lambda \) and \( c \) are small enough. It is instructive to discuss the role of these parameters in turn.

First, a demand for external certification obtains when \( m \) is large. This observation demonstrates the aforementioned pooling effect of external certification. If an individual producer tries to build up his own reputation, he has to ensure that his long run gain from staying honest outweighs the short
run gain from cheating. Clearly, the external certifier is in a similar position. Also he must ensure that his long run gain outweighs the short run one. However, since the external certifier may pool all certification jobs, the long run gain from honest behavior is higher than the long run gain for an individual producer. The more producers, the larger this difference. In contrast, the short gain from cheating is independent of the number of producers. This is because a certifier may collect at most the short run gain of an individual producer rather than all producers together. Hence, one may say that an external certifier pools only the long run gains from staying honest and not the short run gains from cheating. This effect makes it easier for a certifier to sustain a reputation than for an individual producer. As the effect increases with the number of producers $m$, a potential demand for external certification is more likely to exist for larger $m$. Putting it differently, one may say that Proposition 8 identifies benefits from specialization. It is easier for a single institution — the external certifier — to provide a reputation than for many individual ones — the producers.

Second, a demand for certification is established for $\lambda$ small enough. This emphasizes a second beneficial effect from external certification. Since $\lambda$ represent a direct, inverted cost of capture, a lower $\lambda$ makes it more costly for a producer to capture the certifier. Consequently, a smaller $\lambda$ makes honest certification easier to achieve. In contrast, cheating on one’s own reputation does not involve a cost and hence a potential for external supervision exists if $\lambda$ is low. One may interpret this result as an illustration of the argument in Kofman and Lawarrée (1993) that internal supervision is easier to manipulate than external supervision and may therefore lead to a demand for external supervision.

Finally, inequality (4) requires a $c$ small enough. As $c$ represents the cost of certification, this effect is straightforward. Honest certification becomes more costly when $c$ increases and, hence, the long run gain from staying honest decreases. Consequently, it becomes more difficult for an external certifier to maintain an honest reputation. Since, building one’s own reputation is independent of $c$, a demand for certification exists only if $c$ is small enough.
8 Discussion

This paper derives conditions under which reputation is an effective mechanism for external certifiers to resist capture and maintain their honesty. It thereby renders commercial certification markets economically viable. Moreover, the need for reputation may actually trigger a demand for external certification, because a credible reputation is easier to establish when it is concentrated. In addition, honest certification requires high prices that exceed the static monopoly price for low discount rates. It moreover exhibits features of a natural monopoly and represents a technology with increasing returns to scale.

In order to analyze the problem we made a set of simplifying assumptions. First, we assumed a perfect detection technologies of the certifier and also the consumer. This allows a straightforward application of the standard theory of repeated games. If either the certifier’s or the consumers’ detection mechanism is imperfect, consumers cannot determine the certifier’s honesty with certainty. In this case one has to resort to the more complicated theory of repeated games with imperfect public information (e.g. Fudenberg, Levine and Maskin 1994). Second, we assumed that at the beginning of the game certifiers commit for once and for all to a price of certification $p$ instead of choosing a price every period. This assumption is not crucial, because in equilibrium certifiers have no incentive to charge different prices. Indeed, one may even dispense with the assumption that consumers observe prices. For example, one may sustain the equilibrium outcomes of Proposition 4 when consumers believe that the monopolistic certifier sets the price $\hat{p}^m$. In this case, the certifier has no incentive to charge a different price and, hence, beliefs are confirmed in equilibrium. Third, because this paper focuses on the problem of capture rather than one-sided opportunistic behavior, we did not allow the certifier to forge a certification outcome by himself even though he may save the cost $c$ this way. Clearly, in practise this problem may also be an important obstacle to honest certification. However, for $\lambda = 1$ this possibility does not affect the outcome, because taking the payment $p$ and certifying at some false quality $q$ yields the certifier less than a bribing offer $(p, 1)$, which in our capture proof equilibria yields less than staying honest. For $\lambda < 1$ additional problems may arise.
In our setup certification addresses distributive distortions rather than allocative ones. Hence, certification has no positive effect on social welfare and the framework cannot be used to study normative questions. Nevertheless, the advantage of this setup is that it yields a clean analysis and enables us to illustrate the main issues in a clear way. Moreover, the intuition behind our results is general and robust if we extend the framework to address allocative distortions. A straightforward extension that would provide a framework to study normative questions is to introduce moral hazard on part of the producers. That is, rather than assuming that producers are endowed with a fixed quality, they actually choose their unobservable, costly quality. This reasonable extension would give certification a welfare enhancing effect, because it may induce producers to choose higher, more socially efficient qualities. The introduction of quality as a strategic variable would however cloud some aspects of our analysis. Nevertheless, our main qualitative findings of an advantage of concentration and the need for super monopoly pricing would also obtain in such more elaborate models.

Finally, we want to close this paper with a remark concerning the Sarbanes–Oxley Act of 2002 which demands a separation of accounting and consulting services in the US. The Act was introduced after the Enron–Andersen scandal and the separation is meant to reduce the threat of capture in accounting. However, if honest certification is based on the reputation arguments of this paper, the separation may actually exacerbate the threat of capture. The Act reduces the amount of future rents to honest certification and, hence, it pays the certifier less to remain honest. In popular debate this effect of the Act does not seem to have been recognized.

Appendix

Proof of Proposition 1: In any equilibrium in which Assumption 1 holds capture may not take place, since otherwise the beliefs of consumers are not consistent with the behavior of the certifier. Hence, condition (3) must be satisfied for all $b$. This is the case if and only if for all $b \in [p, 1-p]$ it holds

\[(1 + p - b)\lambda b + (b - p)(p - c + \delta V^h(p)) \leq V^h(p),\]  

(5)
where \( V^h(p) = (1 - 2p)(p - c)/(1 - \delta) \). Solving (5) with respect to \( \delta \) yields

\[
\delta \geq \tilde{\delta}(b) \equiv \frac{b\lambda(1 - b + p) - (1 - b - p)(p - c)}{2p(p - c)(b - p) + b\lambda((1 - p) - b + 2p)}.
\]

Consequently, capture does not take place if and only if \( \delta \geq \delta(b) \) for all \( b \in [p, 1 - p] \), i.e. if \( \delta \geq \max_{b \in [p, 1 - p]} \delta(b) \). It holds

\[
\tilde{\delta}'(b) = \frac{(p - c)(1 - 2p)((1 - b + p)^2\lambda + 2p(p - c))}{[b\lambda(1 + p - b) + 2(b - p)(p - c)p]^2} > 0.
\]

Hence, \( \delta(b) \) is increasing in \( b \) and obtains its maximum

\[
\tilde{\delta}(1 - p) \equiv \frac{\lambda(1 - p)}{\lambda(1 - p) + (1 - 2p)(p - c)}
\]

at the corner solution \( b = 1 - p \). It follows that if an equilibrium exists which satisfies Assumption 1, it must hold

\[
\delta > \delta^c(p) \equiv \tilde{\delta}(1 - p).
\]

Q.E.D.

**Proof of Proposition 2:** We first demonstrate convexity of \( \delta^c(p) \). The second derivative of \( \delta^c(p) \) computes as

\[
\frac{\partial^2 \delta(p)}{\partial p^2} = \frac{2\lambda(1 + 2c^2 - 6p + 12p^2 - 4p^3 + c(1 - 6p) + \lambda(1 - c))}{[(p - c)(1 - 2p) + \lambda(1 - p)]^3}
\]

(6)

The denominator is positive, hence the sign of (6) depends on the numerator. Since \( \lambda \geq 0 \) and \( c < 1 \), the numerator is positive if \( 1 + 2c^2 - 6p + 12p^2 - 4p^3 + c(1 - 6p) \) is positive. The expression is quadratic in \( c \) and obtains a minimum at \( c = (6p - 1)/4 \) of \( (1 - 2p)^2(7 - 8p)/8 \) which is positive for \( p \leq 1/2 \). Consequently, also the numerator in (6) is positive and the second derivative of \( \delta^c(p) \) is positive, which implies convexity.

From the convexity of \( \delta^c(p) \) it follows that first order conditions are sufficient for a minimum. Taking first order conditions yields

\[
\delta^* \equiv \frac{\lambda}{3 - 2\sqrt{2 - 2c - 2c + \lambda}}
\]

and obtains for \( p^* = 1 - \sqrt{(1 - c)/2} \). Due to \( c \leq 1/2 \), it follows that \( p^* \in (p^m, 1) \). From the continuity of \( \delta^c(p) \), \( \delta^c(c) = \delta^c(1/2) = 1 \) and the existence
of a minimum $\delta^*$ on $[0, 1]$ it follows that for any $\delta \in (\delta^*, 1]$ that there exist a price such that $\delta^c(p) = \delta$. Due to the convexity of $\delta^c(p)$ it holds for any price $p \in [p_l(\delta), p_h(\delta)]$ that $\delta > \delta(p)$. From Proposition 1 it follows then that truthful certification for price $p$ is sustainable. Q.E.D.

**Proof of Proposition 3:** The proof of Proposition 2 shows $\delta^c(p)$ obtains a minimum at $p^* = 1 - \sqrt{(1 - c)/2}$. Due to $c \leq 1/2$, it holds $p^* > p^m$. Q.E.D.

**Proof of Lemma 2:** For $b \in [p, 1 - p]$ a bribe is accepted with positive probability and yields the principal $V^c(b|p) = (1 - b + p)b\lambda + (p - b)(p - c)(1 - 2\delta p)/(1 - \delta)$. The derivative w.r.t. $b$ is

$$
(1 + p - 2b)\lambda - (p - c)(1 - 2\delta p)/(1 - \delta)
$$

which is linearly decreasing in $b$ and therefore greater than

$$
(1 + p - 2(1 - p))\lambda - (p - c)(1 - 2\delta p)/(1 - \delta).
$$

The derivative of (8) w.r.t. $\delta$ is $(p - c)(1 - 2p)/(1 - \delta)^2 > 0$ and, therefore, (8) is increasing in $\delta$. Since capture proofness implies that $\delta > \delta^c(p)$ it implies that (8) is greater than $(p - c)(1 - 2p)/(1 - \delta)^2$, which is greater than zero. It follows that the derivative $V^c(b|p)$ w.r.t. $b$ itself is larger than zero and attains its maximum at $b = 1 - p$. That is,

$$
V_{\text{max}}^c(p) \equiv V^c(1 - p|p) = \frac{(p - c)(1 - 2p)(1 - 2\delta p) + (2(1 - p)p\lambda)(1 - \delta)}{1 - \delta}.
$$

It holds

$$
\left. \frac{\partial V_{\text{max}}^c}{\partial p} \right|_{p = p^m} = \frac{(1 - 2c)(4\lambda(1 - \delta) - \delta(1 - 2c))}{4(1 - \delta)}.
$$

Expression (9) is decreasing in $\delta$, because the derivative of (9) with respect to $\delta$ is $-(1 - 2c)^2/(4(1 - \delta)^2) < 0$. Since it holds $\delta \geq \delta^c(p^m)$ it follows that (9) is less than $-(1 + 2c)\lambda/2$ and therefore negative. Q.E.D.

**Proof of Proposition 4:** In any equilibrium that satisfies Assumption 2 capture occurs with zero probability, since otherwise any belief that satisfies Assumption 2 is not consistent with the behavior of the certifier. Hence, if certification is to take place, Proposition 1 implies that for any equilibrium
price \( \hat{p}^m \) it holds that \( \delta \geq \delta^c(\hat{p}^m) \). As \( \delta^c(\hat{p}^m) \geq \delta^* \) it follows \( \delta \geq \delta^* \). Moreover, for \( \hat{p}^m \) to be an equilibrium price, it must be optimal for the certifier. Consequently, Assumption 2 is consistent only with an equilibrium price that solves

\[
\max_{\{p : \delta(p) \leq \delta\}} V^h(p)
\]

which implies \( \hat{p}^m = \max\{p^m, p_l(\delta)\} \). To show existence of such an equilibrium take the out-of-equilibrium beliefs \( q^c_i(q^c_i, n^c_i, H_t, p) = 0 \) for \( \delta(p) > \delta \) and \( n^c_i > 0 \) and \( q^c_i(q^c_i, 0, H_t, p) = \hat{p}^m \). These beliefs ensure that any price \( p' \) with \( \delta(p') > \delta \) yields the certifier zero profit, such that \( \hat{p}^m \) is indeed optimal. Q.E.D.

**Proof of Lemma 3:** For a price \( p_l(\delta) \) it holds per definition that \( V^h(p_l(\delta)) = \max_b V(b|p_l(\delta)) \), where \( V^h(p_l(\delta)) = \sum_{i=1}^N \delta_t^i D^h(p_l(\delta))(p_l(\delta) - c) \). Hence, \( V^h(p_l(\delta)) \) is the payoff of a certifier who sets the price \( p_l(\delta) \) and receives the demand from any producer \( q \geq 2p_l(\delta) \) if some of this demand is served by some other certifier, the payoff from honest certification is strictly less than \( V^h(p_l(\delta)) \) such that condition (3) is violated.

Q.E.D.

**Proof of Proposition 5:** Let \( (p^*_1, \ldots, p^*_N) \) represent a vector of equilibrium prices. Define \( \bar{p} = \min\{p^*_i | \delta(p^*_i) \leq \delta\} \). In any equilibrium \( \bar{p} \) exists, because otherwise all certifiers make zero profit and any certifier is better off setting a price \( p_l(\delta) + \varepsilon \) which yields a strictly positive profit. Suppose \( \bar{p} > p_l(\delta) \), then there exists at least one certifier who does not receive the entire demand for certification. This certifier is better off when he undercut the price \( \bar{p} \) by some \( \varepsilon > 0 \). In this case, he is the certifier that offers the lowest price greater than \( \delta \) such that assumption 3 implies that consumers will trust him. Hence, all producers have a strict incentive to perform their certification at this certifier rather than a different one. Consequently, whenever \( \bar{p} > p_l(\delta) \), there exists a certifier who has an incentive to deviate. Thus, in any equilibrium that satisfies Assumption 3 it must hold \( \bar{p} = p_l(\delta) \).

To show existence, define the following beliefs \( q^c_i(q^c_i, 0, H_t, p) = p_l(\delta) \) and \( q^c_i(q^c_i, 1, H_t, p) = q^c_i \) whenever \( q^c_i \in [2p_l, 1] \), \( \{\tau < t | n^c_i = i \wedge q^c_i \neq q_r\} = \emptyset \) and \( p_l > p_l(\delta) \). Moreover, \( q^c_i(q^c_i, 1, H_t, p) = q^c_i \) whenever \( q^c_i \in [2p_l, 1] \), \( \{\tau < t | n^c_i = 1 \wedge q^c_i \neq q_r\} = \emptyset \) and \( p_l = p_l(\delta) \). Otherwise, \( q^c_i(q^c_i, i, H_t, p) = 0 \). These beliefs satisfy Assumption 3. Let \( p_1 = \ldots = p_N = p_l(\delta) \) and let a producer
Let \( q_r \in [2p_l(\delta), 1] \) certify at certifier 1 and let all producers \( q_r \in [0, 2p_l(\delta)) \) offer their goods uncertified. It is straightforward to show that these beliefs and strategies constitute an equilibrium with the outcome that all producers with 
\( q \in [2p_1, 1] \) certify at certifier 1, who certifies honestly at a price \( p_l(\delta) \). Q.E.D.

**Proof of Proposition 6:** Let \( (p_1^*, \ldots, p_N^*) \) represent a vector of equilibrium prices. Define \( \bar{p} = \min\{p_i^* | p_i^* \geq p^*(c^1, c^2)\} \). In any equilibrium \( \bar{p} \) exists, because otherwise either max\(\{p_i^*\} < p_1^* \) such that certifier 1 makes zero profit and is better off with a price \( p_1^* \). Or, max\(\{p_i^*\} \geq p_1^* \) in which case certifier 1 is better off raising its price by some \( \varepsilon > 0 \), since — here the additional restriction on beliefs plays a role — consumers only trust certifier 1 at these prices.

Case 1: Suppose \( p_1^* \geq p^m(c^1) \) and \( \bar{p} > p_1^* \), then according to Proposition 4 the price \( p_1^* \) yields a monopolistic certifier with costs \( c^1 \) the highest payoff. Since in any equilibrium that satisfies Assumption 4 consumers only trust certifier 1 to behave honestly at a price \( p_1^* \), charging this price yields certifier 1 strictly more. Hence, \( \bar{p} > p_1^* \) cannot be an equilibrium price.

Case 2: Suppose \( p_1^* < p^m(c^1) < p_2^* \) and \( \bar{p} > p^m(c^1) \), then according to Proposition 4 the price \( p^m(c^1) \) yields a monopolistic certifier with costs \( c^1 \) the highest payoff. Since in any equilibrium that satisfies Assumption 4 consumers only trust certifier 1 to behave honestly at a price \( p^m(c^1) \), this price yields certifier 1 strictly more. Hence, \( \bar{p} > p^m(c^1) \) cannot be an equilibrium price.

Case 3: Suppose \( p_2^* \leq p^m(c^1) \) and \( \bar{p} > p_2^* \), then either certifier 1 or 2 does not obtain the entire demand of certification. Hence, by a similar argument as in the proof of Proposition 5 \( \bar{p} \) cannot be an equilibrium and in equilibrium it holds \( \bar{p} = p_2^* \). Moreover, in any equilibrium with price \( \bar{p} = p_2^* \) certifier 1 must obtain all demand, since otherwise he is better off undercutting the price \( p_2^* \) by some \( \varepsilon > 0 \). At this price consumers only trust certifier 1 such that he obtains the entire demand. Q.E.D.

**Proof of Lemma 4:** Define \( \Delta(\gamma, c) \equiv p_m(\gamma c) - p_l(\hat{\delta}, c) \). By assumption \( \Delta(\gamma, c^2) > 0 \). Moreover, it holds
\[
\frac{\partial \Delta(\gamma, c)}{\partial c} = -\frac{1}{2} \left( 1 - \gamma + \frac{\hat{\delta}(1 - 2c + \lambda) - \lambda}{\sqrt{\hat{\delta}(1 + 2c) + \lambda(1 - \hat{\delta})}^2 - 8\hat{\delta}(c \hat{\delta} + \lambda - \hat{\delta}\lambda)} \right)
\]
That is, \( \Delta(\gamma, c) \) is decreasing in \( c \) if \( \delta(1 - 2c + \lambda) - \lambda \geq 0 \). This latter condition holds, when \( 1 - 2c + \lambda > 0 \) and \( \delta \geq \lambda/(1 - 2c + \lambda) \). These two inequalities follow from \( \delta^* > 0 \) and \( \delta \geq \delta^* \geq \lambda/(1 - 2c + \lambda) \). Q.E.D.

**Proof of Proposition 7:** The first part of the Proposition follows from applying Proposition 6 to each period \( \tau \) with discount factor \( (1 - \pi)\delta \).

From \( p_t(\delta, 0) = \min\{p|\delta c(p) = \hat{\delta}\} \) it follows

\[
p_{\infty} = p_t(\hat{\delta}, 0) = \frac{\hat{\delta} + \lambda - \hat{\delta}\lambda - \sqrt{-8\hat{\delta}(\lambda - \hat{\delta}\lambda) + (\delta + \lambda - \hat{\delta}\lambda)^2}}{4\hat{\delta}}.
\]

Therefore,

\[
dp_{\infty}/d\pi = \frac{\partial p_{\infty}}{\partial \hat{\delta}} \frac{\partial \hat{\delta}}{\partial \pi} = \frac{\lambda \delta(3 + \lambda) - \lambda + \sqrt{(\hat{\delta}(1 - \lambda) + \lambda)^2 - 8\delta(1 - \hat{\delta})}}{4\hat{\delta} \sqrt{(\hat{\delta}(1 - \lambda) + \lambda)^2 - 8\delta(1 - \hat{\delta})}}.
\]

The expression is positive if \( \delta(3 + \lambda) - \lambda \geq 0 \), which follows from \( \delta \geq \delta^* = \lambda/(3 + \lambda - 2\sqrt{2}) > \lambda/(3 + \lambda) \). Q.E.D.

**References**


